Class Notes, 31415 RF-Communication Circuits

Chapter III

LINEAR ACTIVE TWO-PORTS

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NB 231

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III Linear, Active Two-Ports

Small-signal components and circuits are often characterized by sets of two-port parameters. They may be used for calculating the interaction between the two-port and a surrounding network on a frequency by frequency basis. Practical circuit design problems are often more involved and may include other topics, for instance choosing between alternative components and circuit realizations in view of, for instance, stability or ease of tuning. The concepts and methods that are presented below support that type of work, where we first shall focus on concepts and results that are based directly on two-port parameters. They are measurable and therefore less prone to uncertainty about coverage than analytical network methods based on simplified theoretical device models.

Dealing with RF circuits there are two basic approaches to two-port parameters, either scattering parameters - also called s-parameters - or the traditional admittance, impedance and hybrid types - y, z, h, g - of small-signal parameters. S-parameters relate a set of incident and reflected wave quantities that are defined in terms of the port voltages and currents. They have the advantage of being simple to measure at high frequencies. The traditional parameter types constrain port voltages and currents by terminal conditions that are open or short circuits, and they are difficult to establish in practice at high frequencies. However, the conventional parameter types, especially the y-parameters, are often more informative in presentation of concepts and methods from an electronic circuit design point of view, than are s-parameters. Therefore, we start traditionally by introducing y-parameters and use them to describe basic power gain and stability measures. They are all properties, which keep their interpretation regardless of the small-signal parameter type that was used in the underlying measurements or calculations. Conversion between the different parameter types is a routine task that is included in many instruments and design programs. A excellent, thorough presentation based on s-parameters may be found in Ref.[1]

III-1 Y-Parameter Characterization of Two-Ports

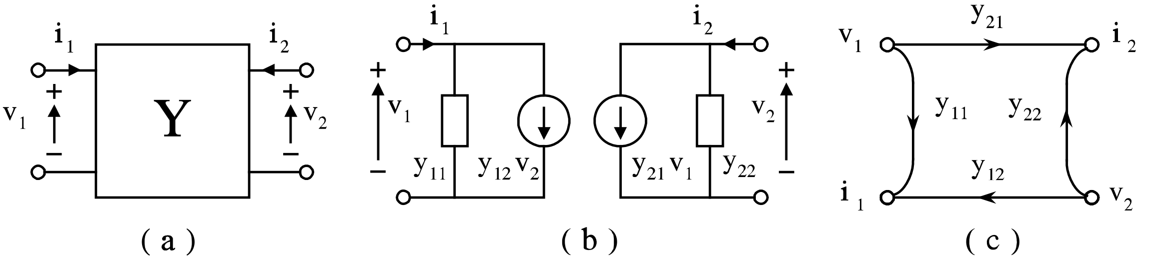


Fig.1 Representation of Y-parameters in (a) block form, (b) the corresponding circuit diagram, and (c) as a signal flow-graph.

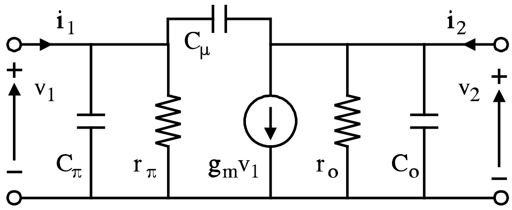
Y-parameters for a two-port are the matrix elements in the linear relationships that express the port currents in terms of the port voltages,

$$\begin{Bmatrix} i_1 \\ i_2 \end{Bmatrix} = \begin{Bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{Bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} g_{11} + jb_{11} & g_{12} + jb_{12} \\ g_{21} + jb_{21} & g_{22} + jb_{22} \end{Bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} , \qquad (1)$$

or in short matrix notation

$$\boldsymbol{i} = \boldsymbol{Y}\,\boldsymbol{v} \ , \qquad \textit{where} \qquad \boldsymbol{i} = \begin{Bmatrix} i_1 \\ i_2 \end{Bmatrix} , \qquad \boldsymbol{v} = \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} . \qquad (2)$$

The last version of the Y-matrix in Eq.(1) shows the elements by real part conductances, g_{ij}, and imaginary part susceptances, b_{ij}. Y-parameters may be visualized in block, equivalent circuit, or signal flow graph forms as indicated by Fig.1. The preference of y-parameters among the traditional parameter types is ascribed to the fact, that linear small-signal RF models for many active devices are of Π shape in an approximation that commonly encompasses all but ultimate high-frequency limits. The Π structure relates directly to a y-parameter description, which gets the advantage of being interpretable in terms of the physical structure of the device, in some cases the parameters may even be traced back to include the bias dependency of the device. A simple example is the transistor equivalent circuit in Fig.2.



$$\boldsymbol{Y}(j\omega) = \begin{Bmatrix} 1/r_\pi + j\omega(C_\pi + C_\mu) & -j\omega C_\mu \\ g_m - j\omega C_\mu & 1/r_o + j\omega(C_o + C_\mu) \end{Bmatrix} \qquad (3)$$

Fig.2 Π-equivalent circuit for transistor.

Had the transistor been a bipolar transistor, the most dominating parameters could be estimated from the basic device relations, cf. refs.[2] chap.7 or [3] p.255ff, p.610ff,

$$g_m = \frac{I_c}{V_t}, \qquad V_t = \frac{kT}{q} \approx 25\,mV, \qquad r_\pi = \frac{\beta_F}{g_m}, \qquad C_\pi > \approx \frac{g_m}{2\pi f_T}. \tag{4}$$

where I_C is the collector DC bias current, β_F is the current gain, and f_T is the cut-off frequency where the extrapolated common emitter transistor current gain equals one. In an initial approximation β_F, f_T and the remaining parameters may be taken as constants.

One way of obtaining y-parameters from a circuit diagram should be noticed from Fig.2 and Eq.(3). Assume short circuited output terminals, i.e. $v_2 = 0$, and apply a unit voltage generator across the input terminals. Then the input and the output port currents i_1, i_2 - both with positive direction towards the two-port - equal y_{11} and y_{21} respectively. Short circuiting the input terminals and applying a unit voltage to the output terminals provides correspondingly y_{12} and y_{22} from i_1 and i_2.

Passive and Active Two-Ports

A two-port that can deliver net-power to a surrounding network at a given frequency is called active at that frequency. The contrast is a passive two-port, which is further classified as being either lossless in the limit case of zero power consumption or strictly passive if it consumes positive net power. Note that we take activity and passivity as frequency dependent properties and include only sinusoidal steady-state behavior. Enlarged scopes are sometimes required in more fundamental considerations and here the reader should consult literature on circuit theory like ref.[4]. Dependence on frequency is commonly not written explicitly below. Unless stated otherwise this should always be assumed.

The group of passive two-ports that contains passive components only, for instance capacitors, resistors, and inductors but no controlled sources, are also reciprocal, which implies y_{12}=y_{21}, cf.[5] chap.5 or [7] sec.4.5. For this group passivity applies at all frequencies. Active two-ports are often linearizations of electron device characteristics around a DC-bias point. The power gain, which is ability of the device to convert DC power to amplified small-signals, depends on frequency. Typically, the power gain of a transistor decreases as frequency raises and eventually it starts consuming net power.

In mathematical terms, the total power consumed by a two-port is expressed by the inner vector product[1] of the terminal voltages and currents,

$$\boldsymbol{P}_{tot} = \boldsymbol{Re}\{\, \boldsymbol{v}^*\boldsymbol{i} \,\} = \frac{1}{2}\,(\, \boldsymbol{v}^*\boldsymbol{i} + \boldsymbol{i}^*\boldsymbol{v} \,) = \frac{1}{2}\,\boldsymbol{v}^*\,(\, \boldsymbol{Y} + \boldsymbol{Y}^* \,)\,\boldsymbol{v}\,. \tag{5}$$

1) The symbol * denotes the Hermitian operation, which transposes vectors and matrices and complex conjugates the elements.

Passivity requires $P_{tot} \geq 0$ for any **v**. It is the same to say that the Hermitian form above must be positive semidefinite or, equivalently, that all principal minors in the y-parameter determinant, which is the set of subdeterminants that can be taken symmetrically around the diagonal, are positive or zero. From the determinant,

$$\det\left[\frac{1}{2}\left(\boldsymbol{Y}+\boldsymbol{Y}^*\right)\right] = \begin{vmatrix} g_{11} & \frac{1}{2}\left(y_{12}+y_{21}^*\right) \\ \frac{1}{2}\left(y_{21}+y_{12}^*\right) & g_{22}\end{vmatrix}, \tag{6}$$

we get three conditions,

$$g_{11} \geq 0\,, \qquad g_{22} \geq 0\,, \qquad g_{11}g_{22} - \frac{1}{4}\left|y_{21}+y_{12}^*\right|^2 \geq 0\,. \tag{7}$$

Had we confined our theory to one ports, the passivity requirement is a non-negative real part of the admittance. The two first requirements above are the one port condition at either port if the opposite port is short-circuited. Introducing the rewriting,

$$y_{21}y_{12} + y_{21}^*y_{12}^* = 2Re\{y_{21}y_{12}\} = 4g_{21}g_{12} - 2Re\{y_{21}^*y_{12}\} = 4g_{21}g_{12} - y_{21}^*y_{12} - y_{21}y_{12}^*, \tag{8}$$

the third condition in Eq.(7) is transformed

$$\begin{aligned} g_{11}g_{22} - \frac{1}{4}\left|y_{21}+y_{12}^*\right|^2 &= g_{11}g_{22} - \frac{1}{4}\left(y_{21}+y_{12}^*\right)\left(y_{21}^*+y_{12}\right) \\ &= g_{11}g_{22} - \frac{1}{4}\left(y_{21}y_{21}^* + y_{12}y_{12}^* + 4\,g_{21}g_{12} - y_{21}^*y_{12} - y_{21}y_{12}^*\right) \\ &= g_{11}g_{22} - g_{12}g_{21} - \frac{1}{4}\left|y_{21}-y_{12}\right|^2 \geq 0 \end{aligned} \tag{9}$$

The last line provides the passivity requirement

$$1 \geq U \geq 0\,, \qquad \textit{where} \qquad U \equiv \frac{\left|y_{21}-y_{12}\right|^2}{4\left(g_{11}g_{22}-g_{12}g_{21}\right)}\,. \tag{10}$$

Here, the boundary to one is a direct consequence of Eq.(9) while the boundary to zero appears because the last term in the equation is nonnegative.

The quantity U is the so-called Mason's Unilateral Power Gain. As the name suggests, U has a wider interpretation than just being an indicator for passivity. It is proven in Appendix III-A that U is invariant to passive, lossless encapsulation of the two-port, i.e. a new two-port build like Fig.3 by encapsulating the original two-port in a passive network of capacitors and inductors, coupled or uncoupled, will give the same U figure. Of particular interest is here the network that makes the new two-port unilateral, which means a resultant zero-valued feedback with $\hat{y}_{12}=0$. In this case U expresses the power gain of the combined

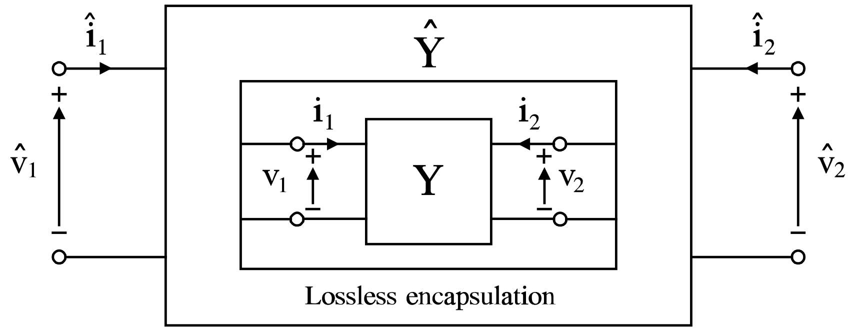


Fig.3 Lossless encapsulation of a two-port. The maximum unilateral power gain U is the same for the intrinsic, Y, and the encapsulated two-port, $\hat{Y}$.

two-port when the source and load admittances maximize the gain by simultaneous conjugated matching at the ports.

Lossless encapsulations includes also device lead substitutions that turn common emitter/source configurations into common base/gate or common collector/drain configurations. Thus, U for an active device does not depend on configuration but remain the same whether a transistor is employed in common emitter, base, or collector, respectively source, gate, or drain configurations.

Considering experimental y-parameters, U commonly decreases with increasing frequency. The frequency where U passes one - equivalently 0 dB - is the frequency where the device turns passive. This frequency is called the maximum frequency of oscillation, f_{max}. The device cannot provide power gain and sustain oscillations - planned or spurious - in any passive embedding above f_{max}. Fig.4 shows an example of U based on experimental transistor data. It is worth noticing that the maximum frequency of oscillation usually is higher than the cut-off frequency f_T that traditionally is used to specify the frequency limitation of transistors.

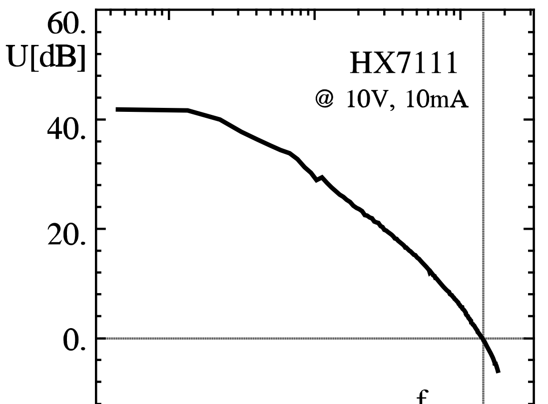


Fig.4 Maximum unilateral power gain U for a bipolar microwave transistor. The data sheets specify f_T= 6 GHz, the maximum frequency of oscillation is f_{max}= 13 GHz

Two-Port Power Gains

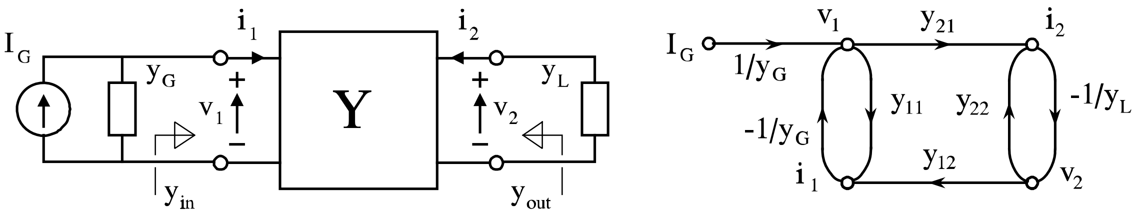


Fig.5 Two-port between external generator and load. The signal flow-graph may be used for calculating gains or input and output admittances.

Active two-ports may be driven and loaded to give net power gain. With a two-port inserted between a generator and a load as shown in Fig.5, we distinguish between three types of power gain[2].

$$\underline{\textit{operating power gain}}\,, \qquad G_p \;\equiv\; \frac{P_L\ :\ \textit{power delivered to the load } Y_L}{P_{in}\ :\ \textit{power consumed at the input port}}\,, \tag{11}$$

$$\underline{\textit{transducer power gain}}\,, \qquad G_{tr} \;\equiv\; \frac{P_L\ :\ \textit{power delivered to the load } Y_L}{P_{av}\ :\ \textit{available generator power}}\,, \tag{12}$$

$$\underline{\textit{available power gain}}\,, \qquad G_{av} \;\equiv\; \frac{P_{out}\ :\ \textit{available power at the output port}}{P_{av}\ :\ \textit{available generator power}}\,. \tag{13}$$

The power gains depend upon the y-parameters and - in different ways - on the generator and load admittances. To see how the gains compare, the basic definitions provide immediately

$$G_{tr} \;\le\; G_p \qquad \textit{and} \qquad G_{tr} \;\le\; G_{av}\,, \tag{14}$$

since, for a given available generator power P_{av} , the input power P_{in} will always be less than or equal to the available power. Correspondingly, the output available power P_{out} is always greater than or equal to the power delivered to the load P_L.

Besides terminating ports, the admittances y_G and y_L determine the input and output admittances of the port through

$$y_{in} = g_{in} + j\,b_{in} = y_{11} - \frac{y_{21}y_{12}}{y_{22} + y_L}\,, \tag{15}$$

$$y_{out} = g_{out} + j\,b_{out} = y_{22} - \frac{y_{21}y_{12}}{y_{11} + y_G}\,. \tag{16}$$

2) Operating power gain is also called simple power gain or just power gain.

These relationships are important for the following power and gain calculations. To see the differences between the power gains, expressions for the various power quantities must first be established. We have directly,

$$P_{in} = |v_1|^2 \, g_{in} \,, \tag{17}$$

where g_{in} is the real part of the input admittance. The voltage gain of the port is

$$A_v = \frac{v_2}{v_1} = \frac{-y_{21}}{y_{22} + y_L} \,, \tag{18}$$

so the power delivered to the load is given by,

$$P_L = |v_2|^2 \, g_L = |v_1|^2 \left| \frac{y_{21}}{y_{22} + y_L} \right|^2 g_L \,, \tag{19}$$

where g_L is the load conductance from $y_L = g_L + jb_L$. The available generator power becomes

$$P_{av} \equiv \frac{|I_G|^2}{4\, g_G} = \frac{|v_1|^2 \, |y_G + y_{in}|^2}{4\, g_G} \,. \tag{20}$$

The last rewriting is due to the fact, that the generator current I_G divides between the generator and the two-port input admittances. The available output power P_{out} is expressed through the output short-circuit current $i_2|_{v_2=0}$,

$$P_{out} = \frac{\left| i_2|_{v_2=0} \right|^2}{4 g_{out}} = \frac{|y_{21}|^2}{4 g_{out}} \left| v_1|_{v_2=0} \right|^2 = \left| \frac{y_{21}}{y_{11} + y_G} \right|^2 \frac{g_G}{g_{out}} \, P_{av} \,, \tag{21}$$

where the last part of (20) was used with the condition that y_{in} reduces to y_{11} when the output port is short-circuited letting $y_L \to \infty$. Now expressions for all the powers in Eq.(13) are established and the gains may be stated explicitly as follows.

$$\underline{\textit{operating power gain}} : \quad G_p \equiv \frac{P_L}{P_{in}} = \left| \frac{y_{21}}{y_{22} + y_L} \right|^2 \frac{g_L}{g_{in}} \,, \tag{22}$$

$$\underline{\textit{transducer power gain}} : \quad G_{tr} \equiv \frac{P_L}{P_{av}} = \left| \frac{y_{21}}{(y_{11} + y_G)(y_{22} + y_L) - y_{21}y_{12}} \right|^2 4\, g_G \, g_L \,, \tag{23}$$

$$\underline{\textit{available power gain}} : \quad G_{av} \equiv \frac{P_{out}}{P_{av}} = \left| \frac{y_{21}}{y_{11} + y_G} \right|^2 \frac{g_G}{g_{out}} \,. \tag{24}$$

For a given two-port, i.e. with known y-parameters, the operating power gain depends only upon the load admittance y_L, the available power gain depends solely upon the generator admittance y_G, whereas the transducer power gain includes the effects of both y_L and y_G. Note that G_p and G_{av} require $g_{in} > 0$ and $g_{out} > 0$ respectively to stay meaningful.

Optimal Power Gain

A natural question to ask at this stage is whether or not it is possible to maximize gains by adjusting the load and generator admittances. It is supposed that y_L and y_G are passive, so they have nonnegative real parts. Maximizing gain is not always possible. If it is, however, we may realize that a admittance pair $y_{L,opt}$, $y_{G,opt}$ that maximizes the transducer gain, simultaneously holds the load that maximizes the simple gain and the generator admittance that maximizes the available gain. To see this, suppose we have chosen a load that optimizes G_p, i.e.

$$y_L = y_{L,opt} \Rightarrow G_p = G_{p,\max} = \left| \frac{y_{21}}{y_{22} + y_{L,opt}} \right|^2 \frac{g_{L,opt}}{g_{in}(y_{L,opt})} \tag{25}$$

where the input admittance is calculated through Eq.(15). If we now adjust the generator admittance to match the input admittance conjugatedly, i.e.

$$y_G = y_{in}^*(y_{L,opt}) \Rightarrow y_G + y_{in} = 2\,\mathbf{Re}\{\, y_{in}(y_{L,opt})\,\} = 2g_{in}(y_{L,opt}) \tag{26}$$

the corresponding transducer gain is seen to be the same as the operating gain from Eq.(25),

$$\begin{aligned} G_{tr} &= \left| \frac{y_{21}}{(y_{in} + y_G)\,(y_{22} + y_L)} \right|^2 4g_{in}g_L = \left| \frac{y_{21}}{2g_{in}(y_{L,opt})\,(y_{22} + y_{L,opt})} \right|^2 4g_{in}(y_{L,opt})g_{L,opt} \\ &= \left| \frac{y_{21}}{y_{22} + y_{L,opt}} \right|^2 \frac{g_{L,opt}}{g_{in}(y_{L,opt})} = G_{p,\max} \ . \end{aligned} \tag{27}$$

Here, the starting version of G_{tr} follows from Eq.(23) with y_{11} substituted through Eq.(15). An inequality in (14) states that the transducer gain is always less than or equal to the operating power gain. When the two are equal and the operating power gain has maximum as above, the transducer gain must be maximum too. With a given available generator power, optimal transducer gain implies furthermore that the output power is maximized and consequently, that the load admittance is conjugatedly matched to the output port. A similar arguing as the one above could be conducted stating from the available power gain and an initial choice of y_G. Then we would get similar conditions for G_{av} and G_{tr}. Therefore, if the power gains of a two-port can be optimized, the same figure applies to all types of gain, i.e.

$$G_{p,\max} = G_{tr,\max} = G_{av,\max} = G_{\max} \ , \tag{28}$$

so there is no need to distinguish between different gain types in the maximum. The gain here is simply called G_{max}. The generator and load admittances that provide maximum gains give conjugated matching at both ports simultaneously. By Eqs.(15),(16) they are related through

$$y^*_{G,opt} = y_{11} - \frac{y_{21}y_{12}}{y_{22} + y_{L,opt}} , \qquad y^*_{L,opt} = y_{22} - \frac{y_{21}y_{12}}{y_{11} + y_{G,opt}} \tag{29}$$

The optimal generator and load admittances could be found by solving the two equations above. Alternatively, one of the gain expressions in Eqs.(22) to (24) could be optimized directly. We shall follow the last approach - the first one is conducted in ref.[6] - by taking outset in the operating power gain and find the value $y_L = y_{L,opt}$ where $\partial G_p / \partial y_L$ becomes zero. Then $y_{G,opt}$ is the complex conjugate of the corresponding two-port input admittance. To ease calculations the following auxiliary variables are introduced

$$y_2 = g_2 + jb_2 \equiv y_{22} + y_L = g_{22} + g_L + jb_{22} + jb_L \tag{30}$$

$$L = P + jQ \equiv y_{21}y_{12} \tag{31}$$

The input conductance and the operating power gain are expressed

$$g_{in} = \frac{g_{11} |\, y_{22} + y_L \,|^2 - y_{12}y_{21}(\, y^*_{22} + y^*_L \,)}{|\, y_{22} + y_L \,|^2} = \frac{g_{11}(\, g_2^2 + b_2^2 \,) - Pg_2 - Qb_2}{(\, g_2^2 + b_2^2 \,)} \tag{32}$$

$$G_p = \frac{P_L}{P_{in}} = \frac{|\, y_{21} \,|^2 \, g_L}{g_{11}\,(\, g_2^2 + b_2^2 \,) - Pg_2 - Qb_2} \tag{33}$$

The steps in the calculations are first to optimize with respect to b_L trough b_2. Next the result $b_{L,opt}$ is inserted into Eq.(33), which now is optimized with respect to g_L trough g_2. Finally the combined result $g_{L,opt}+jb_{L,opt}$ is applied to the first part of Eq.(29) to get the optimal generator admittance. Albeit simple in outline the calculations are somewhat lengthy, so they are detailed separately in Appendix III-B. The following expressions summarize the resultant optimal generator and load admittances,

$$y_{G,opt} = g_{G,opt} + j\, b_{G,opt} , \qquad \begin{cases} g_{G,opt} = g_{11}\, M , \\ b_{Gopt} = \dfrac{Q}{2g_{22}} - b_{11} = \dfrac{Im\{y_{21}y_{12}\}}{2\, g_{22}} - b_{11} , \end{cases} \tag{34}$$

$$y_{L,opt} = g_{L,opt} + j\, b_{L,opt} , \qquad \begin{cases} g_{L,opt} = g_{22}\, M , \\ b_{Lopt} = \dfrac{Q}{2g_{11}} - b_{22} = \dfrac{Im\{y_{21}y_{12}\}}{2\, g_{11}} - b_{22} . \end{cases} \tag{35}$$

The maximum gain that can be obtained with the two-port is expressed,

$$G_{max} = \frac{|\,y_{21}\,|^2}{2\ g_{11}g_{22}\ (\ 1 + M\ )\ -\ Re\{\ y_{21}y_{12}\ \}}\ . \tag{36}$$

Quantity M, which determines the optimal conductances in Eqs.(34),(35) and partly the maximum gain denominator, is given by

$$M \equiv \sqrt{1 - \frac{P}{g_{11}g_{22}} - \left[\frac{Q}{2\ g_{11}g_{22}}\right]^2} = \sqrt{1 - \frac{Re\{\ y_{21}y_{12}\ \}}{g_{11}g_{22}} - \left[\frac{Im\{\ y_{21}y_{12}\ \}}{2\ g_{11}g_{22}}\right]^2}\ . \tag{37}$$

The expression indicates that not all two-ports have optimal gains. Keeping consistency in the conductances that are given by Eqs.(34),(35), M must be real and positive. Therefore, the quantity inside the square root must fulfill the condition

$$1 - \frac{Re\{\ y_{21}y_{12}\ \}}{g_{11}g_{22}} - \left[\frac{Im\{\ y_{21}y_{12}\ \}}{2\ g_{11}g_{22}}\right]^2 > 0 \tag{38}$$

If the Y-parameters of a two-port do not satisfy this criterion, the two-port has no limited optimal power gain and it cannot be simultaneously matched at both ports. We shall see below that it may turn unstable if we try to do so.

Stability of Active Two-Ports

Two-ports that map passive loads into input admittances with negative real parts or passive generator admittances into output admittances with negative real parts may be managed to sustain oscillations. An example is shown in Fig.6, where a passive load y_L by Eq.(15) is supposed to give an input admittance with $g_{in} < 0$ at frequency ω_0 . An admittance Y_G of opposite sign is passive since $g_G > 0$. Connected across the input terminals Y_G gives a total parallel admittance of zero, which is equivalent to an undamped parallel circuit tuned to ω_0. Once initiated to the voltage amplitude v_1, oscillations are sustained while power is delivered to the external conductance g_G. There are no external generators, so this power must come from the two-port.

A two-port through which passive load and generator admittances cannot transform to input and output impedances with negative real parts at a given frequency are said to be absolutely stable at that frequency.[3] If the two-port is not absolutely stable, it called potentially unstable. It should be stressed here that absolute stability will not prevent oscillations if there is a feedback path between the ports outside the two-port. On the other hand, potential

3) If you find the handling of especially limit situations with zero-valued real parts somewhat sloppy, consult ref.[4] for a throughout discussion of the circuit theoretical aspects of these matters.

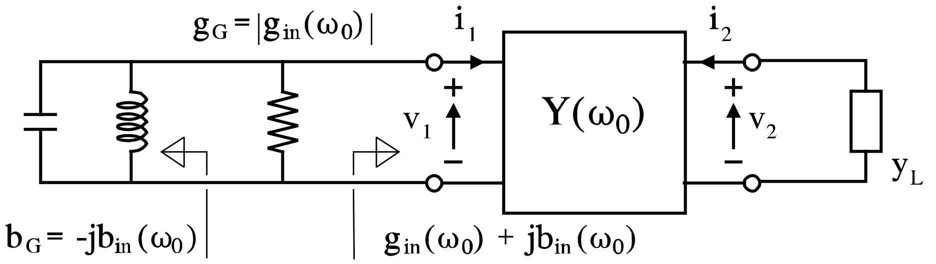


Fig.6 Example of an oscillatory circuit with a two-port that maps the passive admittance y_L into an input admittance having negative real part, $g_{in}(\omega_0)<0$.

instability means the ability of the two-port to oscillate for some - not all - passive terminations. Had we, for instance, in the example of Fig.6 used $g_G > -g_{in}(\omega_0)$ giving a total positive conductance at the input port, any initiated oscillation would decay.

It turns out that criteria for absolute stability become equal to the requirements for a finite maximum power gain of the two-port. Cast in that way we may consider the oscillator setup above as a two-port with infinite gain. To prove the conditions for absolute stability we start introducing the so-called short circuit stability criteria,

$$\underline{\textit{Short-circuit stability}} \; : \; g_{11} = Re\{y_{11}\} > 0, \quad \textit{and} \quad g_{22} = Re\{y_{22}\} > 0. \tag{39}$$

The requirements come about through Eqs.(15),(16). If $y_G \to \infty$ or $y_L \to \infty$ while still being passive, the input and output admittances approach g_{11} and g_{22} respectively, and they must stay positive. With unilateral two-ports where $y_{12}=0$, Eq.(39) holds the only necessities for absolute stability. In the general case the effect of internal feedback must be investigated. To conduct this we take outset in the auxiliary variables and the input conductance expression from Eqs.(30) to (32). However, instead of considering the sign of g_{in} directly, the calculations follow more easily by investigation of quantity T that has the same sign when $g_{11}>0$,

$$\begin{aligned} T \equiv g_{in}\left[\frac{g_2^2 + b_2^2}{g_{11}}\right] &= g_2^2 + b_2^2 - \frac{P\,g_2}{g_{11}} - \frac{Q\,b_2}{g_{11}} \\ &= \left[b_2 - \frac{Q}{2\,g_{11}}\right]^2 + \left[g_2 - \frac{P}{2\,g_{11}}\right]^2 - \frac{P^2 + Q^2}{4g_{11}^2} \end{aligned} \tag{40}$$

$$\textit{where} \quad b_2 = b_{22} + b_L\,, \quad g_2 = g_{22} + g_L$$

The first bracket in the last expression for T has a minimum value of zero as b_2 through b_L may take any value keeping y_L passive. Load susceptance for minimum T therefore becomes

$$b_2 = \frac{Q}{2g_{11}} \quad \Rightarrow \quad b_L = \frac{Q}{2g_{11}} - b_{22} \tag{41}$$

This is seen to be the same susceptance that gave maximum power gains by Eq.(35). The second bracket in T may be zero if

$$g_2 - \frac{P}{2g_{11}} = g_{22} + g_L - \frac{P}{2g_{11}} = 0 \quad \Rightarrow \quad g_L = \frac{P}{2g_{11}} - g_{22} \tag{42}$$

With second bracket becoming zero, T gets negative sign and the two-port is potentially unstable. To secure absolute stability, Eq.(42) must have no solution corresponding to a passive load where $g_L \geq 0$, so as a first condition we get,

$$g_{22} - \frac{P}{2g_{11}} > 0 \ , \tag{43}$$

If this is fulfilled, T will take minimum value for $g_L=0$ where $g_2=g_{22}$. Absolute stability, i.e. T staying positive, now requires

$$\left[g_{22} - \frac{P}{2g_{11}} \right]^2 > \frac{P^2 + Q^2}{4g_{11}^2} \qquad or \qquad M^2 = 1 - \frac{P}{g_{11}g_{22}} - \left[\frac{Q}{2g_{11}g_{22}} \right]^2 > 0. \tag{44}$$

The last version needs $g_{22}>0$ as presupposed. It is equivalent to the condition for maximum power gain stated formerly through Eqs.(37),(38) if the possibility of equality is disregarded.

Investigating for absolute stability may be based on the M quantity above with the short-circuit criteria from Eq.(39). If M is positive, calculations of the port impedances and maximum gain follow directly using the expressions in Eqs.(34) to (36). While the short-circuit conditions are common, criteria equivalent to M are usually found in literature and data sheets. When Eq.(43) is met, the inequality in the first expression of Eq.(44) applies prior to squaring and we have

$$2g_{11}g_{22} - P > \sqrt{P^2 + Q^2} \qquad \Rightarrow \qquad 2g_{11}g_{22} - Re\{ y_{21}y_{12} \} > | y_{21}y_{12} | \tag{45}$$

Together with the short-circuit stability requirements, the last condition is called Llewellyn's absolute stability conditions. A simple reorganization brings it to the form known as Rollet's stability condition[4]

$$\underline{abs.\ stability\ (\ Rollet\ )}: \quad K > 1 \ , \quad K \equiv \frac{2g_{11}g_{22} - Re\{ y_{21}y_{12} \}}{| y_{21}y_{12} |} \tag{46}$$

---

4) Other equivalent conditions that can be derived from Eq.(45) are

$$Linvill\acute{}s\ Stability\ Condition: \ 0 < C < 1 \ , \quad C \equiv \frac{| y_{21}y_{12} |}{2g_{11}g_{22} - Re\{ y_{21}y_{12} \}} \ ,$$

$$Stern\acute{}s\ Stability\ Condition: \ K_{stern} > 1 \ , \quad K_{stern} \equiv \frac{2g_{11}g_{22}}{Re\{ y_{21}y_{12} \} + | y_{21}y_{12} |} \ .$$

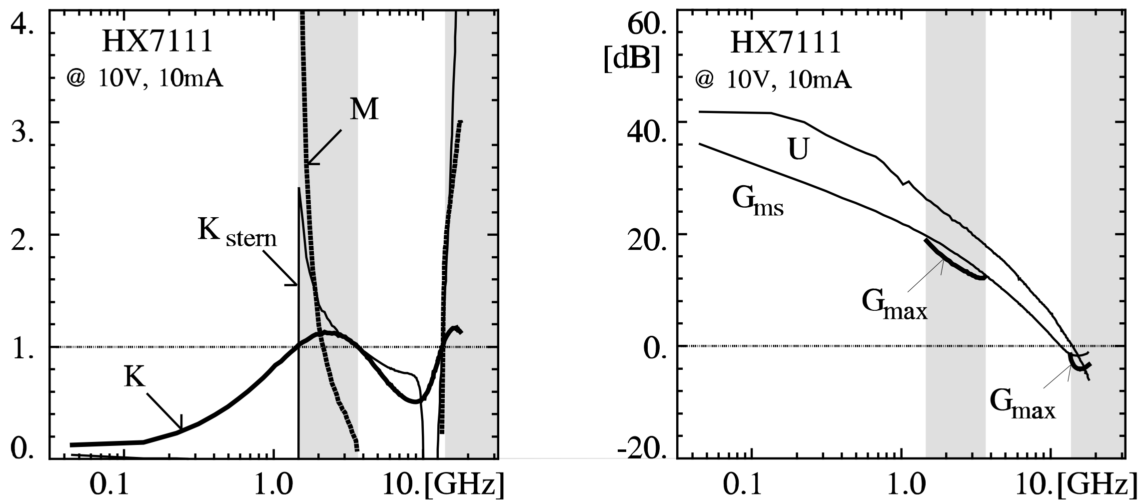


Fig.7 Stability factors and gain functions in a bipolar microwave transistor. Gray zones indicate frequency ranges of absolutely stability. The transistor is passive in the high frequency range.

As demonstrated by Fig.7, Rollet's stability factor K has the advantage of being smooth if it is calculated as a function of frequency. Compared to other criteria, it is therefore well suited for numerical treatment in computer aided design tasks. Using Eq.(44) the relationship between K and M is expressed,

$$4\,g_{11}^2 g_{22}^2\; M^2 \;=\; |\, y_{21} y_{12}\, |^2 \,(\, K^2 - 1 \,) \quad \Rightarrow \quad M = \pm\, \frac{|y_{21} y_{12}|}{2 g_{11} g_{22}} \sqrt{K^2 - 1} \tag{47}$$

Substituting M - only the positive value implies absolute stability - the maximum gain from Eq.(36) may be rewritten,

$$G_{\max} = G_{ms} \frac{1}{K + \sqrt{K^2 - 1}} = G_{ms} \left(K - \sqrt{K^2 - 1} \right), \qquad \textit{where} \quad G_{ms} \equiv \left| \frac{y_{21}}{y_{12}} \right| . \tag{48}$$

The quantity G_{ms} is called the maximum stable gain[5]. In principle it is the maximum gain of a non-unilateral two-port at the boundary to absolute stability with K=1. If the actual two-port is potentially unstable, G_{ms} is the maximum gain that may be obtained if the two-port is extended placing conductances across its ports as shown in Fig.8 below. The y-parameters including extensions become

5) In literature and device data sheets G_{ms} is sometimes denoted MSG (Maximum Stable Gain) and G_{max} may be called MAG (Maximum Available Gain). The latter term is misleading in the sense that if a gain maximum exists, the operating, the transducer, and the available gains were shown to be the same on page 8 above.

$$\tilde{y}_{11} = (g_{11} + g_A) + jb_{11}\,, \qquad \tilde{y}_{22} = (g_{22} + g_B) + jb_{22}\,, \qquad \tilde{y}_{21} = y_{21}\,, \qquad \tilde{y}_{12} = y_{12}\,. \tag{49}$$

This two-port has clearly the same G_{ms} as the original one but the new K-factor, which we get by inserting the new parameters into Eq.(46), may be raised to one or even higher by proper selection of g_A and g_B. If a given two-port without extension is absolutely stable, it has a maximum gain G_{max} and the K-factor exceeds one. According to Eq.(48), G_{max} should fall below G_{ms}, and this is also observed in the absolutely stable frequency band of Fig.7. The G_{ms} parameter has commonly no practical importance in this case although we may think of it as the maximum stable gain that might be attained if negative conductances are connected across the ports up to the point where the resultant K-factor becomes one.

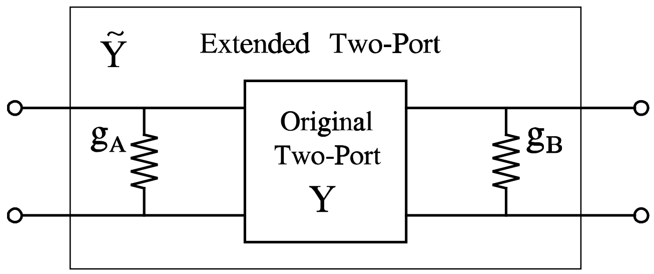


Fig.8 Resistive extension of a two-port. Conductances g_A, g_B control the stability factor K but keep the maximum stable gain G_{ms} unaffected.

Stabilizing Active Two-Ports

RF transistors are commonly potentially unstable in the greater part of the frequency range where they may give net power gain, a fact that is clearly displayed by the experimental data in Fig.7. Absolute stability means that the two-port will stay stable with any passive terminations and with predictable, limited maximum power gain. By contrast, any power gain may be supported, if the two-port is potentially unstable, and we consider the situation of self-sustained oscillations as one of infinite gain. The potentially unstable two-ports may still be useful, if it can be embedded to provide the required, limited gain without start oscillating. There are two main approaches to reach that goal. We may either reduce gain by adjusting matching conditions, possibly by extending with additional resistors, or we may add circuitry to reduce feedback. We shall start considering the first method, where at most we are targeting the maximum stable gain. By the second method - known as neutralization and discussed on page 26 - we may reach gain corresponding to the U function at the costs of more complex circuits.

An obvious way of exerting control over the stability properties of a two-port is shown in Fig.9. The real parts of the generator and load admittances are included into a hypothetical augmented two-port for stability computations. The generator and load conductances presented at the ports should be chosen to fulfil the stability conditions from Eqs.(39) and (46) with y-parameters for the augmented two-port,

$$\hat{y}_{11} = (g_{11} + g_G) + jb_{11}\,, \qquad \hat{y}_{22} = (g_{22} + g_L) + jb_{22}\,, \qquad \hat{y}_{21} = y_{21}\,, \qquad \hat{y}_{12} = y_{12}\,. \tag{50}$$

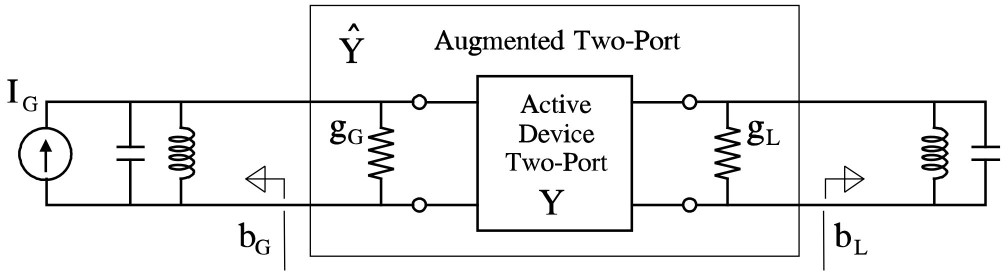


Fig.9 Augmentation of two-port by the generator and load conductances for stability calculations. With K>1 the setup will stay stable for any setting of the susceptances b_G and b_L.

By this technique it is ensured that the terminated two-port stays stable with any generator and load susceptances. To find the pertinent power gains, however, it is not the parameters of the augmented two-port but the parameters of the original one that must be used in Eqs.(22) to (24) together with y_G and y_L for finding the type of gain that is appropriate and defined. The latter question concerns the operational and the available power gains, where it may happen that the terminated two-port gets input or output conductances that are zero or negative, so these gain functions become meaningless.

While the partitioning in Fig.9 was dictated by our stability discussion, practical single stage RF amplifier design commonly gets a structure like Fig.10. Lossless matching networks are inserted at either side of the active two-port to transform given generator and load admittances to the admittances that are required at the ports to the active device. If the device is absolutely stable, either by itself or by the extension method in Fig.8, simultaneous conjugated matching at the device ports provides maximum gain. Since it also implies transfer of available power, i.e. maximum power, to and from the device, and since the matching networks cannot absorb power, the power transfers from the generator and to the load are maximum too, so the outer ports must also be conjugatedly matched. This will be proven formally below.

Designing without simultaneous matching at both device ports, which is a necessity if the device two-port is potentially unstable, the outer generator and load admittances, Y_G and Y_L, may still be transformed through lossless networks to the generator and load admittances

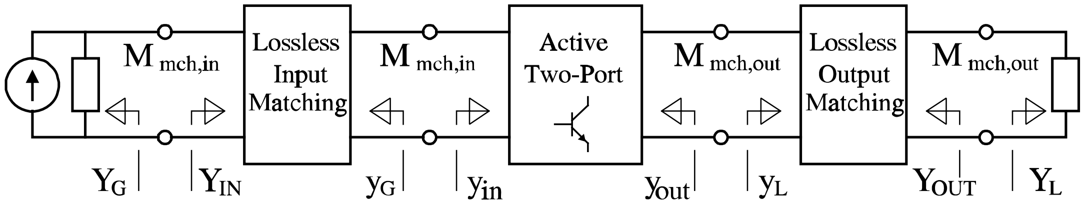


Fig.10 RF-amplifier structure. Matching ratios of input over available powers M_{mch} are the same on either side of lossless networks and equals one with conjugated matching to an absolutely stable device.

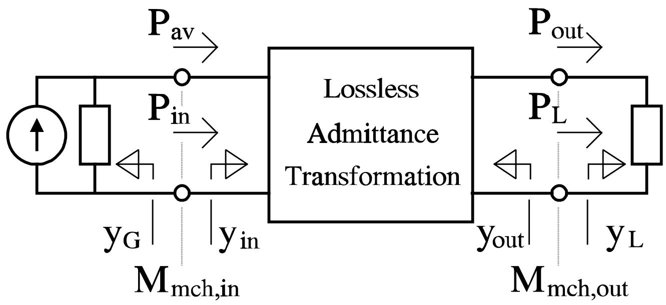


Fig.11 Power flow through an lossless two-port. P_{av} and P_{out} are available, P_{in} and P_L are deposited powers. Notation follows the power gain section, page 6 ff.

that are present at the device ports, y_G and y_L respectively. One design objective is here to get a K-factor in the augmented estimation from Fig.9, which as a minimum requirement exceeds one. In this type of designs it is important to realize that the mismatching around the device two-port pertains to the outer connection ports too.

Introducing a measure of matching, M_{mch}, as the ratio of power delivered to a load over the available power - sometimes called the mismatch factor,[7] - this ratio remains the same at either side of a lossless admittance transforming network. To see this we consider the lossless two-port connection in Fig.11, were we shortly return to the notation in the power gain section on page 6 in order to utilize previous results directly. Before considering constraints set by the lossless two-port, the power expressions from Eqs.(17) through (21) give the matching factors directly in terms of two connected admittances by,

$$M_{mch,in} = \frac{P_{in}}{P_{av}} = \frac{4\, g_G\, g_{in}}{|y_G + y_{in}|^2} \,, \qquad M_{mch,out} = \frac{P_L}{P_{out}} = \frac{4\, g_L\, g_{out}}{|y_L + y_{out}|^2} \,. \tag{51}$$

A reciprocal lossless matching network may be described by y-parameters of susceptances, i.e. purely imaginary components. This may be realized from fulfilling the passivity requirement in Eq.(7) with equal to zero conditions only, so the y-parameters are written

$$y_{11} = j b_{11} \,, \qquad y_{22} = j b_{22} \,, \qquad y_{12} = y_{21} = j b_{21} \,. \tag{52}$$

Substituting this set of parameters into the expression for $y_{in}=g_{in}+jb_{in}$ from Eq.(15) yields,

$$g_{in} = Re\left\{ y_{11} - \frac{y_{12}y_{21}}{y_{22} + y_L} \right\} = \frac{Re\left\{ \left(y_{11}y_{22} - y_{12}y_{21} + y_{11}y_L \right) \left(y_{22}^* + y_L^* \right) \right\}}{|\, y_{22} + y_L|^2}$$
$$= \frac{Re\left\{ \left(b_{21}^2 - b_{11}b_{22} - b_{11}b_L + j b_{11} g_L \right) \left(g_L - j b_L - j b_{22} \right) \right\}}{|\, y_{22} + y_L|^2} = \frac{g_L\, b_{21}^2}{|\, y_{22} + y_L|^2} \,. \tag{53}$$

with load admittance components $y_L=g_L+jb_L$. When this result is inserted into Eq.(22) using $|y_{21}|^2=b_{21}^2$, we get a formal proof of the reasonable result that the operating power gain of a lossless two-port is one.

$$G_p \equiv \frac{P_L}{P_{in}} = \left| \frac{y_{21}}{y_{22} + y_L} \right|^2 \frac{g_L}{g_{in}} \underset{lossless}{=} 1 \ . \tag{54}$$

A similar development starting from y_{out} in Eq.(16) would show that the available power gain through a lossless two-port is one too. By the power gain definitions, Eqs.(11) to (13), the matching ratio at either side of the two-port both become equal to the transducer power gain and thereby also jointly equal,

$$M_{mch,in} = \frac{P_{in}}{P_{av}} = \left.\frac{G_{tr}}{G_p}\right|_{\substack{lossless\\two-port}} = G_{tr} \ , \qquad M_{mch,out} = \frac{P_L}{P_{out}} = \left.\frac{G_{tr}}{G_{av}}\right|_{\substack{lossless\\two-port}} = G_{tr} \ . \tag{55}$$

Thus, the matching ratio stays constant across a lossless two-port whether it is conjugatedly matched with $M_{mch}=1$ or mismatched with ratios below one.[6]

There are situations where matching is an ultimate requirement, for instance set by regulations to equipment employed in common installations. To accomplish this we may resort to the technique in Fig.8 and adjust real parts of y_{11} or y_{22} or both in the active two-port by parallel connecting resistors. The decision of adding to the input, output, or to both conductances may be guided by other concerns. We shall see later that an additional resistive loss at the input may decrease the noise performance of the amplifier. An additional output conductance, on the other hand, may limit the power output capability of the active device.

Even if an active device is absolutely stable, either inherently or by resistive extensions, it might still be useful in a final design to estimate the stability factor for the augmentation encompassing the load and generator admittances. If the device itself is close to being potentially unstable having a K-factor close to one, the factor of the augmentation may indicate how sensitive the complete circuit is with respect to tuning and other parameter variations. The K-factor in the augmented estimation may be used as a degree-of-stability indicator like phase or gain margins in LF designs. This property is discussed in the following two examples, where the first one demonstrates why a K-factor of five or more is a preferable choice in design.

---

6) In microwave literature and data-sheets the grade of mismatch is often expressed by the so-called standing wave ratio, SWR (sometimes VSWR for voltage standing wave ratio). The relationship to M_{mch} is, cf.[7] sec.5.7,

$$SWR \equiv \frac{1+|\Gamma|}{1-|\Gamma|} \ , \qquad where \qquad |\Gamma| = \sqrt{\frac{P_{av}-P_{in}}{P_{av}}} = \sqrt{1 - M_{mch}} \ .$$

Example III-1-1 (degree of stability)

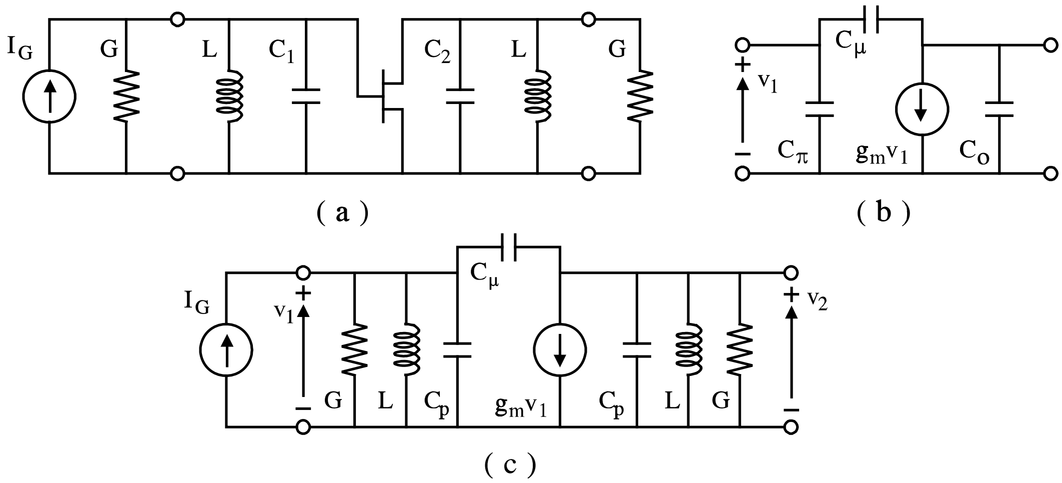


Fig.12 Simple, symmetric narrowband FET amplifier (a) with transistor equivalent circuit in (b). Stability conditions are calculated by the two-port in (c).

To see the K-factor in the role of a degree-of-stability indication we consider the primitive amplifier example in Fig.12. A FET with the simple equivalent circuit in Fig.12b is enclosed between two parallel, equally tuned narrowbanded resonance circuits. The input and output capacitors are supposed to compensate for the transistor input and output capacitances to given a total capacitance of C_p in both circuits, i.e.

$$C_p = C_1 + C_\pi = C_2 + C_o \; . \tag{56}$$

The two-port we consider for stability calculations is shown by Fig.12c. The y-parameter matrix is expressed through $y_p(s)$, which represents the admittance function of the parallel tunings at either side of the transistor

$$\boldsymbol{Y} = \begin{Bmatrix} y_p(j\omega) & -j\omega C_\mu \\ g_m - j\omega C_\mu & y_p(j\omega) \end{Bmatrix} , \quad y_p(j\omega) = j\omega\,(C_p + C_\mu) + G + \frac{1}{j\omega L} \; . \tag{57}$$

The transfer impedance of the amplifier is expressed,

$$\frac{v_2}{I_G} = z_{21} = \frac{-y_{21}}{y_{11}y_{22} - y_{21}y_{12}} = \frac{-y_{21}}{y_p^2 - y_{21}y_{12}} = \frac{-y_{21}}{(\,y_p + \Delta\,)\,(\,y_p - \Delta\,)} \tag{58}$$

$$where \qquad \Delta = \sqrt{y_{21}y_{12}}$$

The first rewriting uses the fact that the Z-parameter matrix is the inverse of the Y-parameter matrix. Around the frequency f_o, where the circuits are tuned, y_p is expressed through the narrowband approximation from chap.II p.16, i.e.

$$y_p(\omega) \approx 2(C_p + C_\mu)(j\omega - s_{p0}), \qquad where$$

$$s_{p0} \approx -\frac{\omega_o}{2Q} + j\omega_0, \qquad \omega_0 = \frac{1}{\sqrt{LC}}, \qquad Q = \frac{\omega_0}{W_{3dB}} = \frac{(C_p + C_{mi})\,\omega_0}{G}. \tag{59}$$

Here s_{p0} is the upper half-plane zero of $y_p(s)$. Had the transistor been unilateral, y_{12}=0, there would have been a double pole at s_{p0} in the amplifier transfer impedance, cf.Eq.(58). The feedback in the transistor changes the picture. The admittance of C_μ is supposed to be much smaller than the transconductance g_m at the center frequency, so we approximate,

$$y_{21} = g_m - j\omega C_\mu \approx g_m, \quad y_{12} = -j\omega C_\mu \approx -j\omega_0 C_\mu, \quad \Rightarrow \quad \Delta = \sqrt{-j}\,\sqrt{g_m C_\mu \omega_0} \tag{60}$$

Simplification of y_{12} to be taken as a constant quantity agrees with our confinement to a narrowband frequency interval around f_0. Under the same assumptions, the stability factor of the two-port is related to Δ through,

$$K \approx \frac{2\,Re\{y_p(\omega_0)\}^2}{|\Delta|^2} = \frac{2G^2}{g_m C_\mu \omega_0} \qquad \Rightarrow \qquad \Delta = \sqrt{-j}\;G\sqrt{\frac{2}{K}} \tag{61}$$

Introduction of this result to the transfer impedance of Eq.(58) shows explicitly the pole-splitting into s_{p1} and s_{p2},

$$\frac{v_2}{I_G} = z_{21}(\omega) \approx \frac{-g_m}{4C^2\,(j\omega - s_{p1})\,(j\omega - s_{p2})}, \qquad where$$

$$s_{p1} = j\omega_0 - \frac{\omega_0}{2Q}\left\{1 - \sqrt{-j}\sqrt{\frac{2}{K}}\right\}, \qquad s_{p2} = j\omega_0 - \frac{\omega_0}{2Q}\left\{1 + \sqrt{-j}\sqrt{\frac{2}{K}}\right\}. \tag{62}$$

Due to the feedback through C_μ, the poles moves along a line in directions of $\mp j^{1/2}$ from the position of the unilateral double-pole in s_{po}. The smaller K value, the greater displacement from s_{po}. If K becomes less than one, it is seen from the pole position sketch in Fig.13(a) that the lower pole moves into the right half of the s-plane and makes the amplifier unstable. So there is full agreement between this traditional circuit analysis stability condition and the two-port concepts we have introduced. The frequency responses with various K-values are given in Fig.13(b), where it is seen that the greater K, the better resemblance to the unilateral limit K→∞. It explains why a K value of more than five is preferable, if the design goal is to approximate an ideal unilateral characteristic.

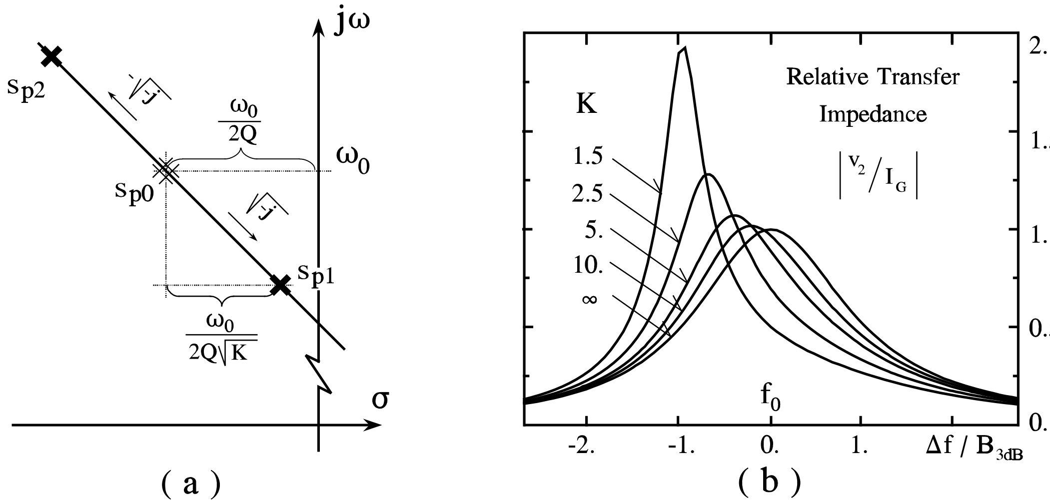


Fig.13 Feedback effects in the amplifier from Fig.12. Pole positions, (a), and transfer characteristics, (b), for various K values corresponding to different C_μ´s.

Example III-1-1 end

Example III-1-2 (amplifier design)

$$\boldsymbol{Y}_{trans} = \begin{Bmatrix} g_{11} + jb_{11} & g_{12} + jb_{12} \\ g_{21} + jb_{21} & g_{22} + jb_{22} \end{Bmatrix} = \begin{Bmatrix} 43.7 + j10.6\ mS & -1.00 - j4.29\ mS \\ -13.9 - j331.\ mS & 1.46 + j15.8\ mS \end{Bmatrix} . \qquad (63)$$

Design a 500 MHz amplifier with the bipolar transistor MRF8372 at V_{CE}=12.5V, I_C= 150mA, where measurements give the y-parameters in Eq.(63). The amplifier must meet specifications,

- center frequency gain, 15 dB
- simultaneous match to 50Ω at both ports
- amplifier must stay stable without load or/and generator connected
- 3dB bandwidth 45MHz in two stage synchronous tuning

Before starting we notice that the transistor data imply, cf. Eq.(46),

$$y_{21}y_{12} = (1.00 + j4.29)(13.9 + j333.) = -1415. + j392.6\,[mS]^2 = 1468.\,[mS]^2 \angle 164.4° \;\Rightarrow$$

$$K_{trans} = \frac{2g_{11}g_{22} - Re\{y_{12}y_{21}\}}{|y_{12}y_{21}|} = \frac{2\cdot 43.7\cdot 1.46 + 1415.}{1468} = 1.051 \; . \qquad (64)$$

Albeit being absolutely stable the transistor is close to the stability bound at one so the requirement of stability without load and source connections is highly sensitive to parameter variations. However, the transistor has a maximum stable gain of

$$G_{ms} = \left| \frac{y_{21}}{y_{12}} \right| = \sqrt{\frac{g_{21}^2 + b_{21}^2}{g_{12}^2 + b_{12}^2}} = \sqrt{\frac{13.9^2 + 331^2}{1.00^2 + 4.21^2}} = 76.56 \sim 18.84 \ [dB] \ , \tag{65}$$

Compared to the gain requirement there is room for enlarging the K-factor by resistive extension. Introducing the ratio of maximum gain - simultaneous matching is required - over the maximum stable gain, Eq.(48) may be solved for the K-factor of an extended two-port,

$$a \equiv \frac{G_{max}}{G_{ms}} : \quad a = K_{ext} - \sqrt{K_{ext}^2 - 1} \quad \Rightarrow \quad K_{ext} = \frac{1}{2} \left(a + \frac{1}{a} \right) . \tag{66}$$

Inserting actual figures provides

$$\begin{aligned} G_{max} = 15\,[dB] \sim 31.62 \quad \Rightarrow \quad a = \frac{31.62}{76.56} = 0.4130 \quad \Rightarrow \\ K_{ext} = \frac{1}{2} \left(0.4130 + 2.421 \right) = 1.417 \ . \end{aligned} \tag{67}$$

In this design we extend the transistor two-port by permanently adding parallel conductance g_B across the collector port as indicated by Fig.8. The K_{ext} value above place us more safely on the proper side regarding stability, in particular if no other admittance are connected across the device ports. The output conductance of the extended two-port is called $g_{22,ext}$, and it may be solved for through the K-factor expression from Eq.(46). We get

$$\begin{aligned} g_{22,ext} = g_{22} + g_B = \frac{K_{ext}\left|y_{12}y_{21}\right| + Re\left\{y_{12}y_{21}\right\}}{2\,g_{11}} = \frac{1.417 \cdot 1468. - 1415.}{2 \cdot 43.7} = 7.610 \ [mS] \ , \\ g_B = g_{22,ext} - g_{22} = 7.61 - 1.46 = 6.15\,[mS] \quad \sim \quad r_B = \frac{1}{g_B} = 163\,[\Omega] \ . \end{aligned} \tag{68}$$

Output conductance $g_{22,ext}$ is the only replacement that is required to find the y-parameter matrix of the extended two-port compared to the original one in Eq.(63). To find the generator and load admittances that imply simultaneous matching to the extended two-port, we start by calculating M from Eq.(47), i.e

$$M = \frac{\left|y_{12}y_{21}\right| \sqrt{K_{ext}^2 - 1}}{2\,g_{11}\,g_{22,ext}} = \frac{1468. \sqrt{1.417^2 - 1}}{2 \cdot 43.7 \cdot 7.61} = 2.216 \ . \tag{69}$$

Now the optimal generator and load admittances may be found using Eqs.(34),(35),

$$\begin{aligned} g_G = g_{11}\, M = 43.7 \cdot 2.216 = 96.84\,[mS] \ , \\ b_G = \frac{Im\left\{y_{12}y_{21}\right\}}{2 g_{22,ext}} - b_{11} = \frac{392.6}{2 \cdot 7.61} - 10.6 = 15.20\,[mS] \ . \end{aligned} \tag{70}$$

$$g_L = g_{22,ext}M = 7.61 \cdot 2.216 = 16.86\,[mS] \ ,$$

$$b_L = \frac{Im\{y_{12}y_{21}\}}{2g_{11}} - b_{22} = \frac{392.6}{2 \cdot 43.7} - 15.8 = -11.31\,[mS] \ . \tag{71}$$

Augmenting by the generator and load conductances, the stability factor of the amplifier under normal operation becomes

$$K_{aug} = \frac{2(g_{11}+g_G)(g_{22,ext}+g_L) - Re\{y_{12}y_{21}\}}{|y_{12}y_{21}|}$$
$$= \frac{2(43.7+96.84)(7.61+16.86)+1415.}{1468} = \frac{6878 + 1415}{1468} = 5.649 \ . \tag{72}$$

As discussed in the previous example, a total stability factor of five or more is a practical design criterion that makes the amplifier relatively insensitive to parameter spreadings. Furthermore it allows us to disregard internal feed-back effects and design the two matching and tuning networks independent of each other.

The results obtained thus far are summarized by Fig.14. The generator and load impedances presented at the ports to the transistor two-port including the shunting resistor R_B are known. They give simultaneous matchings, so their complex conjugated counterparts are the input admittances of the transistor. Therefore, the design may be completed from the point of view, that the two matching networks should transform the extended transistor input and output admittances to the required generator and load values of 50Ω subject to bandwidth constraints. The latter implies that the Q-factors at either side of the transistor are equal, Q_{mc}, and deduced from the specifications by taking into account the gain-bandwidth factor for two stages, cf. Table I, chap.II, p.33,

$$Q_{total} = \frac{f_0}{BW_{3dB}} = \frac{500}{45} = 11.11, \qquad Q_{mc} = Q_{total}GBF_2 = 11.11 \cdot 0.643 = 7.144 \ . \tag{73}$$

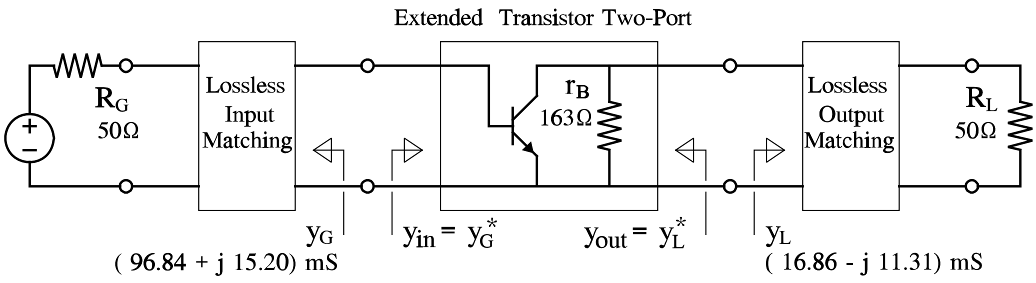


Fig.14 Amplifier principle including the data that are determined from gain and stability requirements.

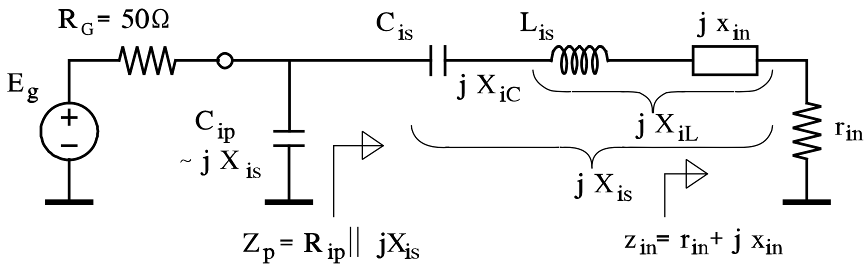


Fig.15 Details in design of the input matching network. z_{in} is the input impedance of the extended transistor when it is matched conjugatedly at the output port.

The low impedance level at the input side of the transistor would give unrealistic component values if we attempted a direct parallel tuning here. A better choice is to raise the impedance level by series connecting a reactance to the transistor input port as indicated by Fig.15, which is equivalent to Example II-4-1 (chap.II p.20.). Converted to series form, the transistor input impedance becomes,

$$z_{in} = r_{in} + j\,x_{in} = y_{in}^{-1} = (\,y_G^*\,)^{-1} = (\,0.09684 - j0.01520\,)^{-1} = 10.08 + j1.580\ [\Omega]. \tag{74}$$

The total series reactance X_{is} must be chosen to convert r_{in} to parallel form R_{pin}=50Ω to match the generator at the amplifier input port. Simultaneously the capacitive parallel reactance X_{ip} must tune out X_{is}. We assume here - and in all subsequent calculations - that we may use series-to-parallel conversions in the simplest form, cf. chap.II,p.20,

$$R_{ip} = R_G = \frac{X_{is}^2}{r_{in}} \Rightarrow X_{is} = \sqrt{R_G\,r_{in}} = \sqrt{50\cdot 10.08} = 22.45\ [\Omega]\ , \qquad C_{ip} = \frac{1}{X_{is}\,\omega} = \frac{1}{22.45\cdot 2\cdot\pi\cdot 500\cdot 10^6} = 14.18\ [pF]\ . \tag{75}$$

Series reactance X_{is} is the net result of an inductive part X_{iL} and a capacitive part X_{iC}. The ratio of X_{iL} - representing storage of magnetic energy - over r_{in} must be twice resultant Q-factor for the input circuit since r_{in} is only half the resistive loss. The remaining loss comes from the generator impedance, had it been transformed in a series connection to the transistor input impedance under matching conditions. Observing that the total inductive reactance includes a slight contribution x_{in} from the transistor input, inductor L_{is} becomes

$$X_{iL} = L_{is}\,\omega + x_{in},\quad 2\,Q_{mc} = \frac{X_{iL}}{r_{in}} \Rightarrow X_{iL} = 2\,Q_{mc}\,r_{in} = 2\cdot 7.144\cdot 10.08 = 144.0\ [\Omega], \qquad L_{is} = \frac{X_{iL} - x_{in}}{\omega} = \frac{144.0 - 1.58}{2\cdot\pi\cdot 500\cdot 10^6} = 45.34\ [nH]\ . \tag{76}$$

The series capacitance C_{is} must reduce the inductive reactance to X_{is} as assumed for impedance transformation, so the input circuit design is completed by

$$X_{iC} = \frac{-1}{C_{is}\,\omega} = X_{is} - X_{iL} \quad \Rightarrow$$
$$C_{is} = \frac{1}{\omega\,(X_{iL} - X_{is})} = \frac{1}{2\pi \cdot 500 \cdot 10^{6}(144.0 - 22.45)} = 2.618\,[pF]\,. \tag{77}$$

At the output side neither direct parallel nor direct series tunings give reasonable component values, so instead the Π-structure in Fig.16(a) is chosen. To get component values we transform the circuit to a resonance circuit in two steps as shown by Fig.16(b) and (c). The two series contributions in the last form must equal each other when we have impedance matching between their parallel forms, $1/g_{out}$ and R_L=Z_L=50Ω. The matching requirement give

$$r_{os} = \frac{X_{op}^2}{g_{out}^{-1}} = \frac{g_{out}}{C_{op}^2\,\omega^2} = r_{Ls} = \frac{X_{Lp}^2}{R_L} = \frac{1}{R_L\,C_{Lp}^2\,\omega^2} \quad \Rightarrow$$
$$\frac{C_{op}}{C_{Lp}} = \sqrt{g_{out}R_L} = \sqrt{0.01686 \cdot 50} = 0.9181\,. \tag{78}$$

Imposing bandwidth requirements on the circuit in Fig.16(c) provides,

$$Q_{mc} = -\frac{X_{op} + X_{Lp}}{2\,r_{os}} = \frac{C_{op}^2\,\omega^2}{2\,g_{out}}\left(\frac{1}{C_{op}\,\omega} + \frac{1}{C_{Lp}\,\omega}\right) = \frac{C_{op}\,\omega}{2\,g_{out}}\left(1 + \frac{C_{op}}{C_{Lp}}\right) \quad \Rightarrow$$
$$C_{op} = \frac{2\,Q_{mc}\,g_{out}}{\omega\left(1 + C_{op}/C_{Lp}\right)} = \frac{2 \cdot 7.144 \cdot 0.01686}{2\pi \cdot 500 \cdot 10^{6}\,(1 + 0.9181)} = 39.99\,[pF]\ , \tag{79}$$
$$C_{Lp} = \frac{C_{op}}{0.9181} = 43.56\,[pF]$$

The capacitor C_{op} in the parallel to series transformation includes the output capacitance of the transistor. The component C_{opp} to be added in the circuit becomes

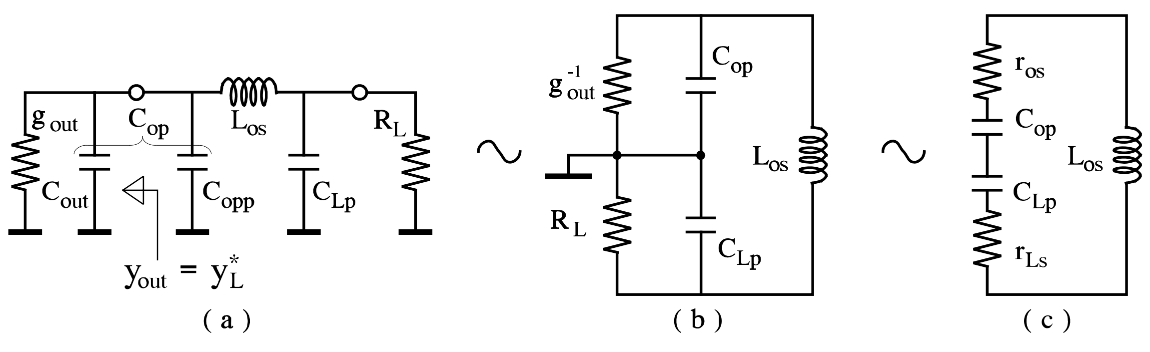


Fig.16 Steps in the design of the output matching network. y_{out} is the output admittance of the extended transistor two-port when the input is conjugatedly matched.

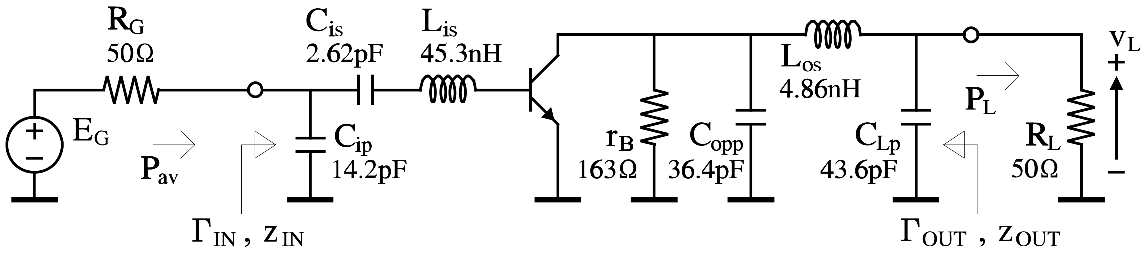


Fig.17 Amplifier functional diagram.

$$C_{opp} = C_{op} - \frac{b_{out}}{\omega} = 39.99\,[pF] - \frac{0.01131}{2\pi \cdot 500 \cdot 10^6} = 39.99 - 3.60 = 36.39\,[pF] \;. \quad (80)$$

Tuning is the last requirement to the output circuit, and it determines the series inductor L_{os},

$$C_{tot} = \frac{C_{op}C_{Lp}}{C_{op}+C_{Lp}} = \frac{36.99 \cdot 43.56}{36.99 + 43.56} = 20.85[pF] \quad \Rightarrow$$

$$L_{os} = \frac{1}{C_{tot}\,\omega^2} = \frac{1}{20.85 \cdot 10^{-12}\,(\,2\pi \cdot 500 \cdot 10^6)^2} = 4.860\,[nH] \;. \quad (81)$$

The equivalent circuit for the amplifier, which summarizes the results above, is shown in Fig.17, while Fig.18 presents simulated responses for the circuit, where

$$G_{tr} = \frac{P_L}{P_{av}} = \frac{v_L^2}{E_G^2/4} \;, \qquad \Gamma_{IN} = \frac{z_{IN} - Z_0}{z_{IN} + Z_0} \;, \qquad \Gamma_{OUT} = \frac{z_{OUT} - Z_0}{z_{OUT} + Z_0} \;, \qquad Z_0 = 50\,[\Omega]\,. \quad (82)$$

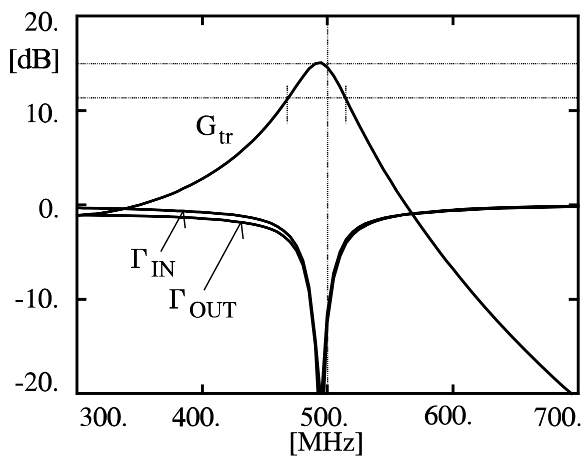


Fig.18 Simulation of the amplifier from Fig.17. The transistor was accounted for by an accurate circuit model extracted from experimental data

As seen, we get the expected gain and bandwidth and nearly correct center frequency. The reflection coefficients with respect to 50Ω have minima that clearly indicate matching at the center frequency. Despite simplifying assumptions - by disregarding feedback and using series parallel transformation in simplest form - the design above is rather precise. It provides a good starting point for subsequent fine tunings either physically using trimmer capacitors or in computer optimizations, a fact that mainly owes to the possibility in this design to satisfy specifications with a high K-factor in the augmented stability estimation.

Example III-1-2 end

Neutralization

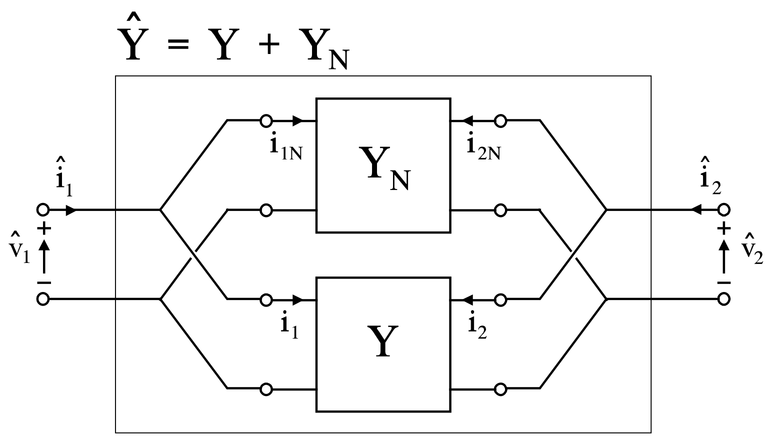


Fig.19 Parallel connection of two-ports. The resultant Y-matrix is the matrix sum of matrices for the parallelled two-ports.

The only requirement for absolute stability in unilateral two-ports, i.e. two-ports with $y_{12=0}$, is the short-circuit conditions in Eq.(39). One mean of keeping the gain high with a given device is therefore to add external circuitry that counteracts the internal feedback. The principle for doing this is suggested by Fig.19. Two two-ports connected in parallel get a total Y-parameter matrix that is the sum of the individual two-ports. To see this recall that Y-parameters have the port voltages as driving variables, and they are common for the parallelled ports. The dependent variables are the currents, and they are added at both sides. Consequently, corresponding y-matrix elements add to give the resultant element,

$$\begin{aligned} \hat{y}_{11} &= y_{11} + y_{11,N} \ , \qquad & \hat{y}_{12} &= y_{12} + y_{12,N} \ , \\ \hat{y}_{21} &= y_{21} + y_{21,N} \ , \qquad & \hat{y}_{22} &= y_{22} + y_{22,N} \ . \end{aligned} \tag{83}$$

To make the resultant two-port unilateral is called neutralization, which requires

$$\underline{\textit{Neutralization}} \ : \quad \hat{y}_{12} = 0 \ , \quad \Rightarrow \quad y_{12,N} = - \, y_{12} \ . \tag{84}$$

A unilateral two-port has M=1 in Eqs.(34) to (37), so to get optimum we must arrange conjugate matching to y_{11} and y_{22} at the input and the output port respectively. We get

$$y_{G,opt} = \hat{y}_{11}^{*} \,, \qquad y_{L,opt} = \hat{y}_{22}^{*} \,, \qquad G_{\max} = \frac{|\,\hat{y}_{21}\,|^2}{4\hat{g}_{11}\hat{g}_{22}} \,. \tag{85}$$

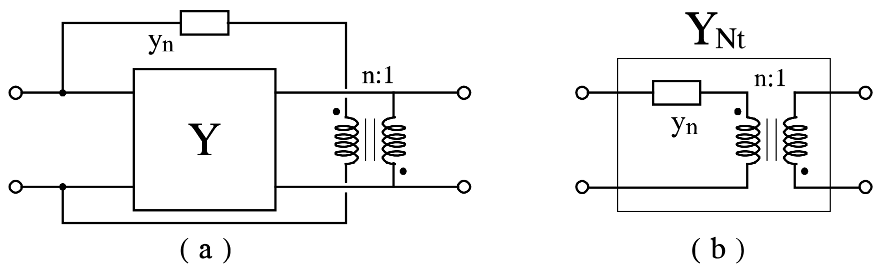


Fig.20 Broadband neutralization with transformer. Neutralizing condition y_{12}=-ny_n.

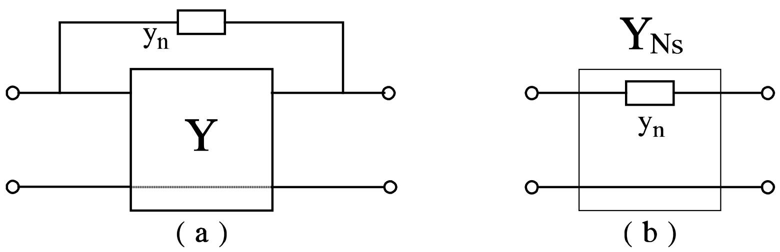


Fig.21 Narrowband neutralization of two-port. Neutralizing condition y_{12}=y_n.

Two common neutralizing principles are shown in Fig.20 and Fig.21, where the y-parameters to be parallelled are

$$\boldsymbol{Y}_{Nt} = \begin{Bmatrix} y_n & ny_n \\ ny_n & n^2 y_n \end{Bmatrix} \,, \qquad \boldsymbol{Y}_{Ns} = \begin{Bmatrix} y_n & -y_n \\ -y_n & y_n \end{Bmatrix} \,. \tag{86}$$

In the last case the added two-port includes a common ground for the ports, a property that also must apply to the neutralized two-port. However, two-ports to be neutralized represent often active three-terminal devices and have common ground inherently. Fig.22 illustrates neutralizations around a transistor with Π-type equivalent circuit. In case the two-port to be neutralized is of Π-type, where y_{12}=-y_{μ}, the transformer coupling in Fig.20 has the capability of covering a broad frequency range. Here the counteracting network contains a scaled version of y_{μ},

$$y_n = -y_{12} = \frac{1}{n} y_{\mu} \tag{87}$$

Remember that neutralization also changes the other parameters. In the present case to, cf. Eqs.(83) and (86),

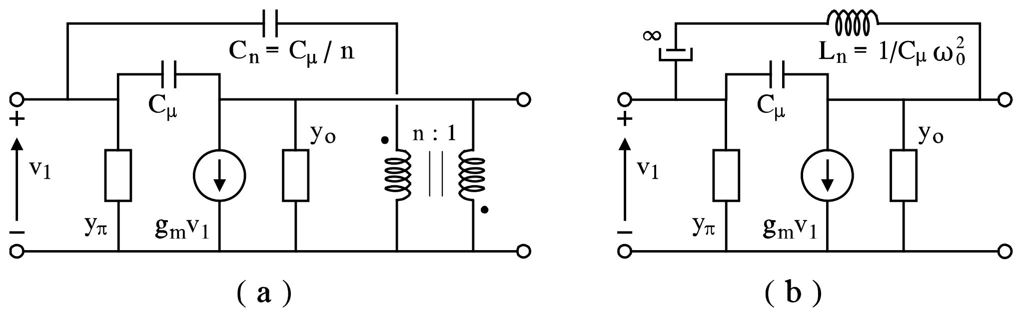


Fig.22 Neutralization of transistor feedback capacitance. A coupling capacitor in (b) is to remind that bias separation between input and output is commonly required.

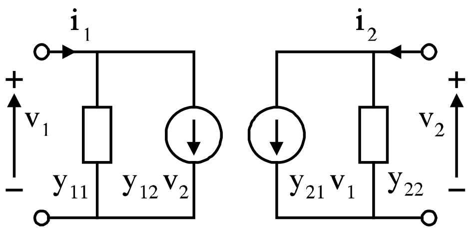
$$\hat{y}_{11} = y_{11} + \frac{y_\mu}{n}, \qquad \hat{y}_{21} = y_{21} + y_{my}, \qquad \hat{y}_{22} = y_{22} + n\, y_\mu \;. \tag{88}$$

The second and simpler method in Fig.21 requires that y_n is the negative of y_{12}. Typically an inductive y_n tunes out a capacitive device feedback at the center frequency of a narrowband amplifier.

Exact neutralization of active devices may require adjustable components to encounter parameter spreading. Setting the resultant y_{12} exactly to zero - for instance by the lossless technique from Appendix III-A - is sometimes called unilateralization and distinguished from a less demanding requirement, where the feedback is reduced sufficiently to guarantee stable operation. Besides the neutralizing techniques considered in this section, an alternative approach for reducing feed-back is to use two devices in cascode coupling. With identical devices - often supplied in a single package for discrete realizations - this method significantly reduces feedback while keeping the forward transfer data practically unchanged.

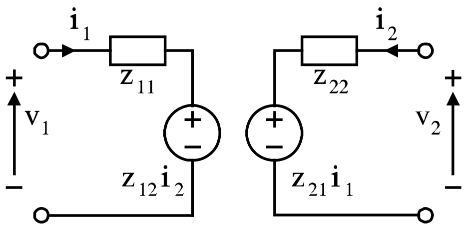
Generalization to Z, H, and G Two-port Parameters

Properties like stability and power gain should be independent of the parameter type chosen to represent the two-port. Four types of small-signal parameters that impose constraints directly between port voltages and currents are summarized below by Fig.23 to Fig.26 and Eqs.(89) to (92). The elements in the **G** matrix should be distinguished from the real part conductances in the **Y** matrix. Conversions between the different types are summarized by Eqs. (95) to (98). While it follows directly from Eqs.(89) to (92) that **Y** and **Z** or **H** and **G** are the inverses of each other, conversions between pure admittances or impedances to the hybrid forms must be deduced separately. Consider as an example the **Y** to **H** conversion where input current i_1 must replace input voltage v_1 as independent variable. The



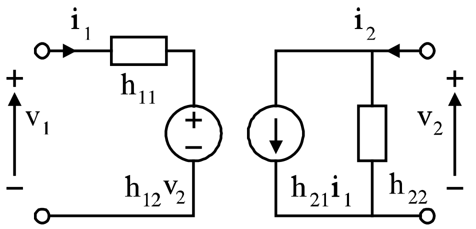
$$\begin{Bmatrix} i_1 \\ i_2 \end{Bmatrix} = \begin{Bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{Bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} \tag{89}$$

Fig.23 Y-parameter two-port



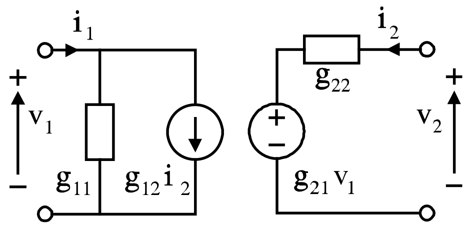
$$\begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{Bmatrix} \begin{Bmatrix} i_1 \\ i_2 \end{Bmatrix} \tag{90}$$

Fig.24 Z-parameter two-port



$$\begin{Bmatrix} v_1 \\ i_2 \end{Bmatrix} = \begin{Bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{Bmatrix} \begin{Bmatrix} i_1 \\ v_2 \end{Bmatrix} \tag{91}$$

Fig.25 H-parameter two-port



$$\begin{Bmatrix} i_1 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{Bmatrix} \begin{Bmatrix} v_1 \\ i_2 \end{Bmatrix} \tag{92}$$

Fig.26 G-parameter two-port

substitution may be visualized by connecting a current generator with impedance Z_G as shown in Fig.27. The flow graph based on y-parameters provides the transformations in the limit of Z_S approaching infinity where the two-port input current i_1 equals the generator current I_G. We get

$$\left.\frac{v_1}{I_G}\right|_{v_2=0} = \frac{Z_G}{1 + y_{11} Z_G} \underset{Z_G \to \infty}{=} \frac{1}{y_{11}} = h_{11}, \qquad \left.\frac{v_1}{v_2}\right|_{I_G=0} = \frac{-y_{12} Z_G}{1 + y_{11} Z_G} \underset{Z_G \to \infty}{=} \frac{-y_{12}}{y_{11}} = h_{12}, \tag{93}$$

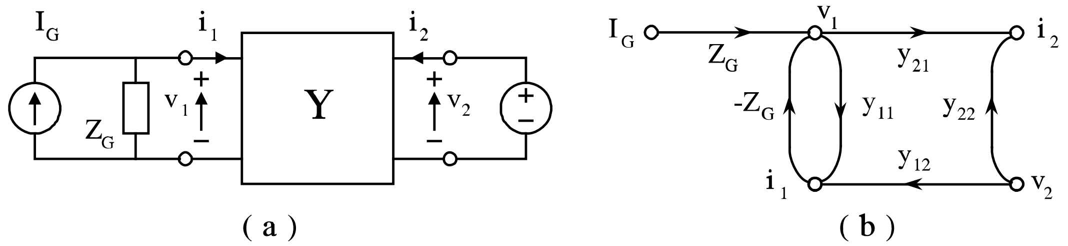


Fig.27 Generator setup and signal flow-graph for y-parameter to h-parameter conversions letting $Z_G \to \infty$.

$$\left.\frac{i_2}{I_G}\right|_{v_2=0} = \frac{Z_G y_{21}}{1+y_{11}Z_G} \underset{Z_G\to\infty}{=} \frac{y_{21}}{y_{11}} = h_{21},$$

$$\left.\frac{i_2}{v_2}\right|_{I_G=0} = \frac{y_{22}(1+y_{11}Z_G)-y_{12}y_{21}Z_G}{1+y_{11}Z_G} \underset{Z_G\to\infty}{=} \frac{\Delta y}{y_{11}} = h_{22} \qquad (94)$$

where Δy is the determinant of the **Y** matrix. Similar arrangements apply to the other conversions. They are all summarized below in Eqs.(95) to (98), where in addition Δz, Δh, and Δg are the determinants of the **Z**, **H**, and **G** matrices respectively.

$$\boldsymbol{Y} = \begin{bmatrix} \dfrac{z_{22}}{\Delta z} & \dfrac{-z_{12}}{\Delta z} \\ \dfrac{-z_{21}}{\Delta z} & \dfrac{z_{11}}{\Delta z} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{h_{11}} & \dfrac{-h_{12}}{h_{11}} \\ \dfrac{h_{21}}{h_{11}} & \dfrac{\Delta h}{h_{11}} \end{bmatrix} = \begin{bmatrix} \dfrac{\Delta g}{g_{22}} & \dfrac{g_{12}}{g_{22}} \\ \dfrac{-g_{21}}{g_{22}} & \dfrac{1}{g_{22}} \end{bmatrix}, \qquad (95)$$

$$\boldsymbol{Z} = \begin{bmatrix} \dfrac{y_{22}}{\Delta y} & \dfrac{-y_{12}}{\Delta y} \\ \dfrac{-y_{21}}{\Delta y} & \dfrac{y_{11}}{\Delta y} \end{bmatrix} = \begin{bmatrix} \dfrac{\Delta h}{h_{22}} & \dfrac{h_{12}}{h_{22}} \\ \dfrac{-h_{21}}{h_{22}} & \dfrac{1}{h_{22}} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{g_{11}} & \dfrac{-g_{12}}{g_{11}} \\ \dfrac{g_{21}}{g_{11}} & \dfrac{\Delta g}{g_{11}} \end{bmatrix}, \qquad (96)$$

$$\boldsymbol{H} = \begin{bmatrix} \dfrac{1}{y_{11}} & \dfrac{-y_{12}}{y_{11}} \\ \dfrac{y_{21}}{y_{11}} & \dfrac{\Delta y}{y_{11}} \end{bmatrix} = \begin{bmatrix} \dfrac{\Delta z}{z_{22}} & \dfrac{z_{12}}{z_{22}} \\ \dfrac{-z_{21}}{z_{22}} & \dfrac{1}{z_{22}} \end{bmatrix} = \begin{bmatrix} \dfrac{g_{22}}{\Delta g} & \dfrac{-g_{12}}{\Delta g} \\ \dfrac{-g_{21}}{\Delta g} & \dfrac{g_{11}}{\Delta g} \end{bmatrix}, \qquad (97)$$

$$\boldsymbol{G} = \begin{bmatrix} \dfrac{\Delta y}{y_{22}} & \dfrac{y_{12}}{y_{22}} \\ \dfrac{-y_{21}}{y_{22}} & \dfrac{1}{y_{22}} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{z_{11}} & \dfrac{-z_{12}}{z_{11}} \\ \dfrac{z_{21}}{z_{11}} & \dfrac{\Delta z}{z_{11}} \end{bmatrix} = \begin{bmatrix} \dfrac{h_{22}}{\Delta h} & \dfrac{-h_{12}}{\Delta h} \\ \dfrac{-h_{21}}{\Delta h} & \dfrac{h_{11}}{\Delta h} \end{bmatrix}. \qquad (98)$$

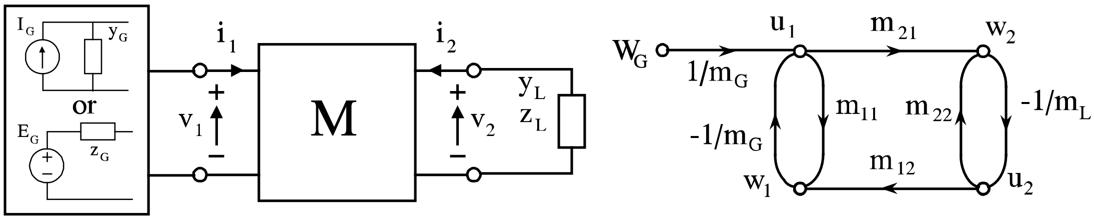


Fig.28 Two-port between external generator of either Norton or Thevenin type and load. M may represent Y, Z, H, and G parameters.

A driven and loaded two-port characterized by any of these sets may be represented by the diagram or flow graph in Fig.28 using one of the substitution rows in Table I. Inserting y-parameters we get the diagram and flow graph of Fig.5 that was basis for most of the previous results. To express gain and stability properties we need expressions for input and output powers to and from the two-port in conjunction with the input and output admittances or impedances. Using any set of parameters from Table I, the input and output impedances and/or admittances are derived from the flow graph to yield

$$m_{in} = m_{11} - \frac{m_{21}m_{12}}{m_{22} + m_L} , \qquad m_{out} = m_{22} - \frac{m_{21}m_{12}}{m_{11} + m_G} . \tag{99}$$

The various types of powers become

$$P_{in} = Re\{ u_1\, w_1^* \} , \qquad P_L = - Re\{ u_2 w_2^* \} ,$$
$$P_{av} = \frac{|\, W_G \,|^2}{4 Re\{ m_G \}} , \qquad P_{out} = \frac{\left|\, w_2 \,|_{u_2 = 0} \right|^2}{4 Re\{ m_{out} \}} . \tag{100}$$

It is seen that we get starting expressions for all types of parameters of the same structures than those that were used with y-parameters starting from Eqs.(17) to (21). Therefore, any other parameter set from the table would develop similarly and provide equivalent expressions. Thus, most gain and stability quantities are unchanged if all equivalent terms from another parameter set are substituted. As an example, stability of a two-port in h-parameters

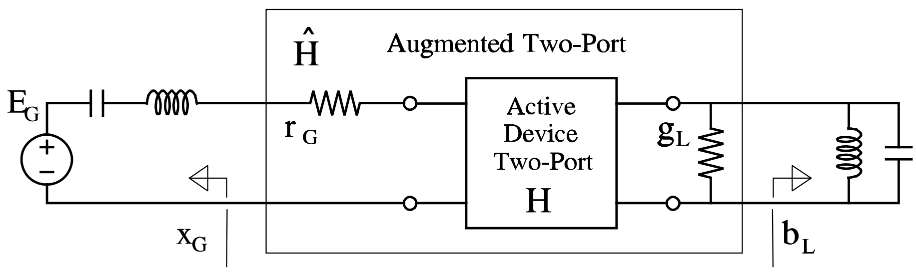


Fig.29 Augmentation of two-port i H-parameters for stability calculations. If K>1 the setup will stay stable with all reactances x_G and susceptances b_L.

Table I Substitution scheme for parameters and variables in Fig.28. The term immittance is used for either impedance or admittance.

Parameter matrix	Generator		Load	Two-port variables	
	drive	immittance	immittance	independent	dependent
	W_g	m_G	m_L	u_1, u_2	w_1, w_2
Y	I_g	y_G	y_L	v_1, v_2	i_1, i_2
Z	V_g	z_G	z_L	i_1, i_2	v_1, v_2
H	V_g	z_G	y_L	i_1, v_2	v_1, i_2
G	I_g	y_G	z_L	v_1, i_2	i_1, v_2

may be secured by stability estimations on the augmentation in Fig.29 and the stability factor similar to Eq.(46),

$$K = \frac{2\left(Re\{h_{11}\} + r_G\right)\left(Re\{h_{22}\} + g_L\right) - Re\{h_{21}h_{12}\}}{|h_{21}h_{12}|} \tag{101}$$

The corresponding transducer power gain becomes, cf. Eq.(23),

$$G_{tr} = \frac{P_L}{P_{av}} = \left|\frac{h_{21}}{\left(h_{11} + z_G\right)\left(h_{22} + y_L\right) - h_{21}h_{12}}\right|^2 4\ Re\{z_G\}\ Re\{y_L\}\ . \tag{102}$$

The only exception from the rule above concerns maximum unilateral power gain U that was introduced by Eq.(10). It was not derived from power considerations involving generator and load circuits but solely from the power consumption of the two-port. Direct insertion of parameter translations from Eq.(95) give

$$U\,\big|_Y = \frac{|y_{21} - y_{12}|^2}{4\left(g_{11}g_{22} - g_{12}g_{21}\right)}\ , \qquad U\,\big|_Z = \frac{|z_{21} - z_{12}|^2}{4\left(r_{11}r_{22} - r_{12}r_{21}\right)}. \tag{103}$$

$$U\,\big|_H = \frac{|h_{21} + h_{12}|^2}{4\left(Re\{h_{11}\}Re\{h_{22}\} + Im\{h_{12}\}Im\{h_{21}\}\right)}\ ,$$
$$U\,\big|_G = \frac{|g_{21} + g_{12}|^2}{4\left(Re\{g_{11}\}Re\{g_{22}\} + Im\{g_{12}\}Im\{g_{21}\}\right)}\ . \tag{104}$$

where the complex g elements in the last expression should not be confused with the real-part conductances in the Y parameters of the first expression.

APPENDIX III-A Properties of the Unilateral Power Gain

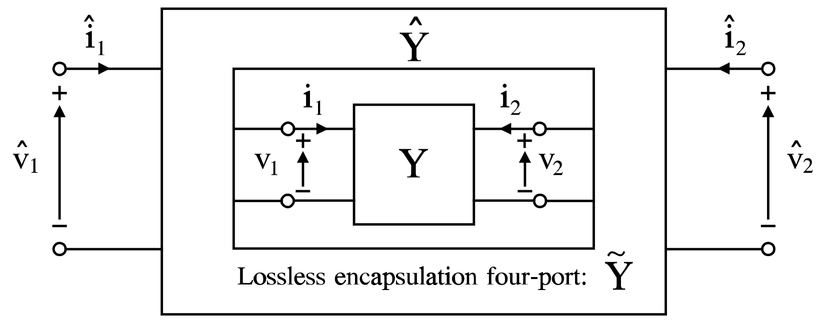


Fig.30 Lossless encapsulation of a two-port. A lossless, reciprocal four-port Ỹ embeds the original two-port, Y, to the resultant two-port, Ŷ.

The appendix gives proofs of the results that are cited regarding the U function, Mason's unilateral power gain. First it is shown that U is invariant with respect to lossless, reciprocal encapsulation, second that - as the name suggests - U actually is the power gain of a two-port in the particular lossless embedding that makes the resultant $\hat{y}_{12}$ zero. To prepare for the first part, the U function is rewritten,

$$U = \frac{|y_{21} - y_{12}|^2}{4(g_{11}g_{22} - g_{12}g_{21})} = \frac{|\det(\boldsymbol{Y} - \boldsymbol{Y}^t)|}{\det(\boldsymbol{Y} + \overline{\boldsymbol{Y}})} . \qquad (105)$$

Equivalency of the two forms may be seen by direct insertion of matrix components. It should be noted that superscript "t" here stands for matrix transposition without complex conjugation, while the overline stands for element by element complex conjugation without matrix transposition.

$$\det\,(\boldsymbol{Y}-\boldsymbol{Y}^t) = \det\begin{Bmatrix} y_{11}-y_{11} & y_{12}-y_{21} \\ y_{21}-y_{12} & y_{22}-y_{22} \end{Bmatrix} = \left[y_{21}-y_{12}\right]^2. \qquad (106)$$

$$\det(\boldsymbol{Y}+\overline{\boldsymbol{Y}}) = \det\begin{Bmatrix} y_{11}+y_{11}^* & y_{12}+y_{12}^* \\ y_{21}+y_{21}^* & y_{22}+y_{22}^* \end{Bmatrix} = \det\begin{Bmatrix} 2g_{11} & 2g_{12} \\ 2g_{21} & 2g_{22} \end{Bmatrix} = 4\left[g_{11}g_{22}-g_{12}g_{21}\right] \qquad (107)$$

Fig.30 above shows a two-port in lossless encapsulation. Y-parameters of the original two-port are in the **Y** matrix while **Ŷ** holds the y-parameters of the embedded two-port. The task to be undertaken is to show that

$$U\Big|_{\hat{Y}} = \frac{|\det(\hat{\boldsymbol{Y}} - \hat{\boldsymbol{Y}}^t)|}{\det(\hat{\boldsymbol{Y}} + \overline{\hat{\boldsymbol{Y}}})} = \frac{|\det(\boldsymbol{Y} - \boldsymbol{Y}^t)|}{\det(\boldsymbol{Y} + \overline{\boldsymbol{Y}})} = U\Big|_{Y} . \qquad (108)$$

The connection between the two sets of two-port parameters is the lossless embedding four-port. The two sets of two-port currents and voltage vectors

$$\hat{\boldsymbol{i}} = \begin{Bmatrix} \hat{i}_1 \\ \hat{i}_2 \end{Bmatrix} , \quad \boldsymbol{i} = \begin{Bmatrix} i_1 \\ i_2 \end{Bmatrix} , \quad \hat{\boldsymbol{v}} = \begin{Bmatrix} \hat{v}_1 \\ \hat{v}_2 \end{Bmatrix} , \quad \boldsymbol{v} = \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} , \tag{109}$$

are constrained by the admittance matrix of the encapsulation matrix. It is a 4×4 matrix, but we organize it in four 2×2 submatrices corresponding to the external and internal two-port connections. As y-parameters are defined with reference directions into the port-circuits, there must be a sign shift in the part of the four-port currents that are in common with the original two-port. The port conditions for the lossless four-port and the original two-port are

$$\begin{Bmatrix} \hat{\boldsymbol{i}} \\ -\boldsymbol{i} \end{Bmatrix} = \begin{Bmatrix} \tilde{\boldsymbol{Y}}_{11} & \tilde{\boldsymbol{Y}}_{12} \\ \tilde{\boldsymbol{Y}}_{21} & \tilde{\boldsymbol{Y}}_{22} \end{Bmatrix} \begin{Bmatrix} \hat{\boldsymbol{v}} \\ \boldsymbol{v} \end{Bmatrix} \quad \textit{where} \quad \boldsymbol{i} = \boldsymbol{Y}\,\boldsymbol{v} . \tag{110}$$

The last row in the first matrix equation is used to express the internal voltage pair from **v** in terms of the external voltage pair in **v̂**, so the internal voltages may be eliminate through the upper part of the matrix equation, i.e.

$$-\boldsymbol{i} = \boldsymbol{Y}\boldsymbol{v} = \tilde{\boldsymbol{Y}}_{21}\tilde{\boldsymbol{v}} + \tilde{\boldsymbol{Y}}_{22}\boldsymbol{v} \quad \Rightarrow \quad \boldsymbol{v} = -\left(\boldsymbol{Y} + \tilde{\boldsymbol{Y}}_{22}\right)^{-1}\tilde{\boldsymbol{Y}}_{21}\tilde{\boldsymbol{v}} \tag{111}$$

$$\hat{\boldsymbol{i}} = \tilde{\boldsymbol{Y}}_{11}\tilde{\boldsymbol{v}} + \tilde{\boldsymbol{Y}}_{12}\boldsymbol{v} = \left[\,\tilde{\boldsymbol{Y}}_{11} - \tilde{\boldsymbol{Y}}_{12}\left(\boldsymbol{Y} + \tilde{\boldsymbol{Y}}_{22}\right)^{-1}\tilde{\boldsymbol{Y}}_{21}\right]\tilde{\boldsymbol{v}} . \tag{112}$$

Thereby, the encapsulated two-port gets the y-parameter matrix,

$$\hat{\boldsymbol{Y}} = \left[\,\tilde{\boldsymbol{Y}}_{11} - \tilde{\boldsymbol{Y}}_{12}\left(\boldsymbol{Y} + \tilde{\boldsymbol{Y}}_{22}\right)^{-1}\tilde{\boldsymbol{Y}}_{21}\right] . \tag{113}$$

Losslessness of the encapsulation network is expressed by letting all submatrices be purely imaginary. This is emphasized by representing them by real-valued susceptance matrices, $\mathbf{B}_{11}$ through $\mathbf{B}_{22}$. Furthermore, it is required that the lossless encapsulation network is reciprocal. This would be the case if the network is build from passive, lossless components, i.e. capacitors, inductors or ideal transmission lines. The above conditions are imposed through

$$\begin{gathered} \tilde{\boldsymbol{Y}}_{11} = j\boldsymbol{B}_{11} : \ \boldsymbol{B}_{11} = \boldsymbol{B}_{11}^{t} , \qquad \tilde{\boldsymbol{Y}}_{22} = j\boldsymbol{B}_{22} : \ \boldsymbol{B}_{22} = \boldsymbol{B}_{22}^{t} \\ \tilde{\boldsymbol{Y}}_{12} = j\boldsymbol{B}_{12} , \quad \tilde{\boldsymbol{Y}}_{21} = j\boldsymbol{B}_{21} : \quad \boldsymbol{B}_{12} = \boldsymbol{B}_{21}^{t} . \end{gathered} \tag{114}$$

The matrix to be inverted inside the y-parameters from Eq.(113) gets significance below. For short we call it **W** and observe that under the assumptions above, and when it is operated upon similarly to the numerator and denominator of the original U equation, it provides directly

$$W \equiv Y + \tilde{Y}_{22} = Y + jB_{22} \quad \Rightarrow \quad \left\{ \begin{array}{l} W + \overline{W} = Y + \overline{Y} \,, \\ W^t - W = Y^t - Y \,. \end{array} \right. \tag{115}$$

Introducing **W** through Eq.(113) into the matrices, which are contained in the numerator and the denominator determinants of the U function for the encapsulated two-port, gives

$$\hat{Y} - \hat{Y}^t = B_{11} + B_{12}\,W^{-1}B_{12}^t - B_{11} - B_{12}^{tt}\,W^{-1t}B_{12}^t = B_{12}\,W^{-1}(\,W^t - W\,)\,W^{-1t}B_{12}^t. \tag{116}$$

$$\hat{Y} + \overline{\hat{Y}} = B_{11} + B_{12}\,W^{-1}B_{12}^t - B_{11} + B_{12}\,\overline{W^{-1}}\,B_{12}^t = B_{12}\,W^{-1}(\,\overline{W} + W\,)\,\overline{W^{-1}}\,B_{12}^t. \tag{117}$$

The determinants themselves now become

$$\left|\,\det(\,\hat{Y} - \hat{Y}^t\,)\,\right| = \frac{\det^2 B_{12}}{|\det W|^2}\,\left|\det(\,Y - Y^t\,)\right|\,, \qquad \det\left(\hat{Y} + \overline{\hat{Y}}\right) = \frac{\det^2 B_{12}}{|\det W|^2}\,\det(\,Y + \overline{Y}\,)\,. \tag{118}$$

Since the leading ratio at the two right hand sides are identical, we have proved Eq.(108), which implies that the U function is invariant to lossless, reciprocal encapsulation.

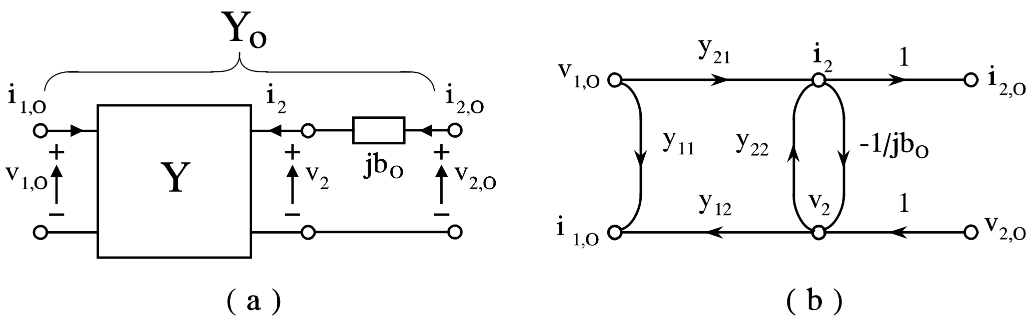


Fig.31 First step towards lossless encapsulation to make an unilateral two-port.

To show that the U function may be interpreted as the maximum power gain of the two-port in a lossless encapsulation, which makes the combined circuit unilateral, we proceed by first demonstrating one method of getting zero-valued reverse admittance. The encapsulation is made in two steps. First a susceptance b_o is placed in series with the output port. The resultant y-parameters, $\mathbf{Y}_o$, are found either from the flow graph in Fig.31 or directly from node-equations to yield,

$$Y_o = \frac{1}{1 + \dfrac{y_{22}}{jb_o}} \left\{ \begin{array}{cc} y_{11} - \dfrac{y_{11}y_{22} - y_{12}y_{21}}{jb_o} & y_{12} \\ y_{21} & y_{22} \end{array} \right\} = \left\{ \begin{array}{cc} \dfrac{y_{11}(y_{22} + jb_o) - y_{12}y_{21}}{y_{22} + jb_o} & \dfrac{y_{12}jb_o}{y_{22} + jb_o} \\ \dfrac{y_{21}jb_o}{y_{22} + jb_o} & \dfrac{y_{22}jb_o}{y_{22} + jb_o} \end{array} \right\}. \tag{119}$$

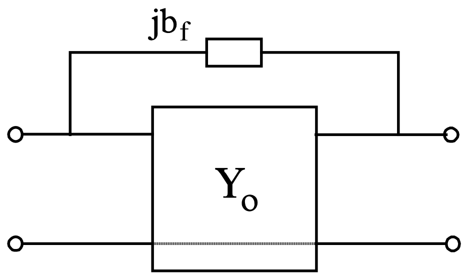


Fig.32 Final step in lossless unilateralization

The purpose of this step is to turn the resultant $y_{o,12}$ into a pure susceptance, which may be canceled by a subsequent parallel connection of the opposite susceptance jb_f as sketched in Fig.32. The technique is discussed in the section on neutralization on page 27 and implies that there is a common ground between the ports. The y-parameters after encapsulation by both jb_o and j_{bf} are given by

$$\hat{\boldsymbol{Y}} = \begin{Bmatrix} \dfrac{y_{11}(y_{22}+jb_o) - y_{12}y_{21}}{y_{22}+jb_o} + jb_f & \dfrac{y_{12}jb_o}{y_{22}+jb_o} - jb_f \\ \dfrac{y_{21}jb_o}{y_{22}+jb_o} - jb_f & \dfrac{y_{22}jb_o}{y_{22}+jb_o} + jb_f \end{Bmatrix} . \tag{120}$$

Enforcing the zero feed-back requirement $\hat{y}_{12}$=0 for real and imaginary parts gives the requirement on b_o and b_f to encapsulate for making an unilateral two-port,

$$(g_{12} + jb_{12})\, jb_o = jb_f\,(g_{22} + jb_{22} + jb_o) ,$$

$$\left.\begin{aligned} \underline{real} &: \; b_{12}b_o = b_f b_{22} + b_f b_o \\ \underline{imag.} &: \; g_{12}b_o = b_f g_{22} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} b_f &= \left(b_{12} - \frac{g_{12}}{g_{22}} b_{22} \right) , \\ b_o &= \left(\frac{g_{22}}{g_{12}} b_{12} - b_{22} \right) . \end{aligned}\right. \tag{121}$$

With a unilateral two-port where $\hat{y}_{12}$=0, the U function becomes

$$U \Big|_{\hat{y}_{12}=0} = \frac{|\,\hat{y}_{21}\,|^2}{4\,\hat{g}_{11}\,\hat{g}_{22}} = G_{\max} . \tag{122}$$

The last equation is recognized as the maximum power gain from Eq.(85) for a two-port with no feedback, i.e. a unilateral one. Recall, that it is not required to find the y-parameters of the two-port embedded for unilateralization to get U. A y-parameter set derived from the original two-port in any lossless encapsulation will do, due to the invariance of the U function.

APPENDIX III-B Conditions for Optimal Power Gain

The appendix details the calculations that led to expressions for the generator and load admittances $y_{G,opt}$, $y_{L,opt}$ in Eqs.(34) through (37). The outset is the expression for operating power gain from Eq.(33) that may be rewritten

$$G_p = \frac{1}{N_1} \mid y_{21} \mid^2 g_L \,, \qquad with \qquad N_1 = g_{11} \left(g_2^2 + b_2^2 \right) - Pg_2 - Qb_2 \tag{123}$$

Here g_2, b_2, P, and Q are the auxiliary variables defined by Eqs.(30),(31) but repeated here for convenience,

$$y_2 = g_2 + jb_2 \equiv y_{22} + y_L = g_{22} + g_L + jb_{22} + jb_L \tag{124}$$

$$L = P + jQ \equiv y_{21} y_{12} \tag{125}$$

If the susceptance b_2 varies, Eq.(123) provides

$$\frac{\partial G_p}{\partial b_2} = \frac{\mid y_{21} \mid^2 g_L}{N_1^2} \left(2b_2\, g_{11} - Q \right) = 0 \quad \Rightarrow \qquad b_{2,opt} = \frac{Q}{2g_{11}} \quad \Rightarrow \quad b_{L,opt} = \frac{Q}{2g_{11}} - b_{11} \,. \tag{126}$$

which is the susceptance part of Eq.(35). Conveying this result to the denominator N_1 of Eq.(123) we get a new denominator called N_2, where

$$N_2 = g_{11} \left(g_2^2 + \frac{Q^2}{4g_{11}^2} \right) - Pg_2 - \frac{Q^2}{2g_{11}} = g_{11} g_2^2 - Pg_2 - \frac{Q^2}{4g_{11}} \tag{127}$$

The gain is now expressed,

$$G_p = \frac{1}{N_2} \mid y_{21} \mid^2 g_L = \frac{1}{N_2} \mid y_{21} \mid^2 (g_2 - g_{22}) \tag{128}$$

Differentiating with respect to g_2 gives

$$\left. \frac{\partial G_p}{\partial g_2} \right|_{b_{L,opt}} = \frac{- \mid y_{21} \mid^2}{N_2^2} \left[N_2 - (g_2 - g_{22}) (2g_{11} g_2 - P) \right] = 0 \quad \Rightarrow \qquad g_2^2 - 2g_{22} g_2 + P \frac{g_{22}}{g_{11}} + \frac{Q^2}{4g_{11}^2} = 0. \tag{129}$$

The last equation has the solutions

$$g_{2,opt} = g_{L,opt} + g_{22} = g_{22} \pm g_{22} \sqrt{1 - \frac{P}{g_{11}g_{22}} - \left[\frac{Q}{2g_{11}g_{22}} \right]^2} = g_{22} \, (1 + M) \qquad (130)$$

$$\Rightarrow \quad g_{L,opt} = g_{22} \, M$$

where M denotes the square root that was previously given by Eq.(37). The final result is the optimal load conductance from Eq.(35). Note that only the solution adding terms is used because it gives a passive generator admittance $g_{L,opt} \geq 0$.

If the optimal conditions apply, the first part of (129) is equivalent to

$$N_2 = g_{L,opt} \left(2 \, g_{11} \, g_{2,opt} - P \right) = g_{L,opt} \left(2 \, g_{11}g_{22}(1 + M) - P \right) \qquad (131)$$

and the optimal power gain becomes

$$G_{p,\max} = G_{\max} = \frac{1}{N_2} \, | \, y_{21} \, |^2 \, g_{L,opt} = \frac{| \, y_{21} \, |^2}{2 \, g_{11}g_{22} \, (1 + M) - Re\left\{ y_{21}y_{12} \right\}} \qquad (132)$$

This expression is the one shown by Eq.(36).

To calculate the optimal source admittance we start from Eq.(32) using the N's from Eqs.(123), (127), and (131),

$$g_{G,opt} = g_{in}\Big|_{Y_{L,opt}} = \left. \frac{N_1}{g_2^2 + b_2^2} \right|_{y_{L,opt}} = \left. \frac{N_2}{g_2^2 + \dfrac{Q^2}{4g_{11}^2}} \right|_{g_{L,opt}} = \frac{g_{22}M \left(2 \, g_{11}g_{2,opt} - P \right)}{g_{2,opt}^2 + \dfrac{Q^2}{4g_{11}^2}}$$

$$= g_{11} \, M \left[\frac{2g_{22}g_{2,opt} - P \, \dfrac{g_{22}}{g_{11}}}{g_{2,opt}^2 + \dfrac{Q^2}{4g_{11}^2}} \right] = g_{11} \, M \, , \qquad (133)$$

Unity of the bracket in the lower line is a consequence of the last optimal condition in (129). The corresponding optimal generator susceptance is calculated directly from Eq.(29) through

$$b_{G,opt} = -Im\left\{ y_{in}\Big|_{y_{L,opt}} \right\} = -b_{11} + Im\left\{ \frac{y_{21}y_{12}}{y_{2,opt}} \right\} = - \, b_{11} + \frac{Q}{2g_{22}} \qquad (134)$$

The last rewriting is based upon the conjugated matching at the output port, $y_2 = 2g_{22}$, which is inherent to optimal gain.

Note that the derivations in this appendix require $g_{11}>0$, $g_{22}>0$, conditions that later on are shown to apply also if the two-port is absolutely stable.

Problems

P.III-1

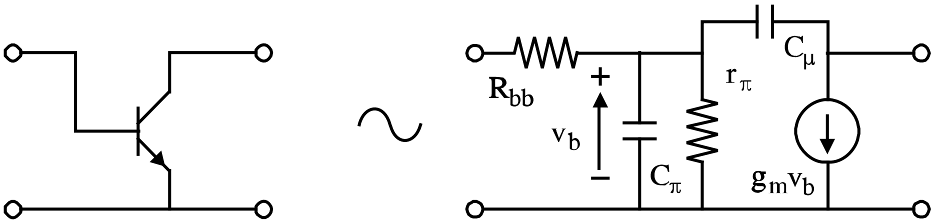


Fig.33 Simple hybrid Π equivalent circuit for bipolar transistor.

Show that the maximum frequency of oscillation for the bipolar transistor in Fig.33 is approximated

$$f_{max} \approx \sqrt{\frac{f_T}{8\pi R_{bb} C_\mu}} \ , \qquad C_\pi \gg C_\mu \ . \tag{135}$$

It is assumed that $f_{max} \gg f_T/\beta$, the so-called β cut-off frequency, and that the feedback capacitance is much smaller than the input capacitance. f_T is the cut-off frequency of the transistor.

Numerical example: R_{bb}=10Ω, C_μ=0.28pF, β=90, and f_T=6GHz.

P.III-2

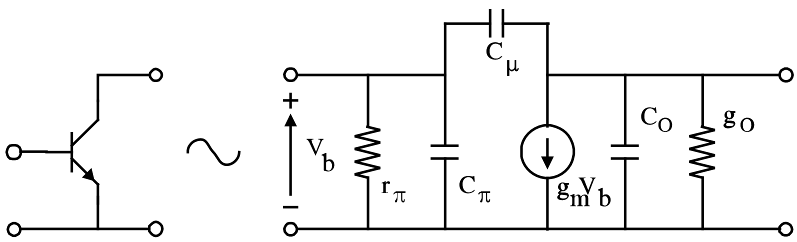


Fig.34

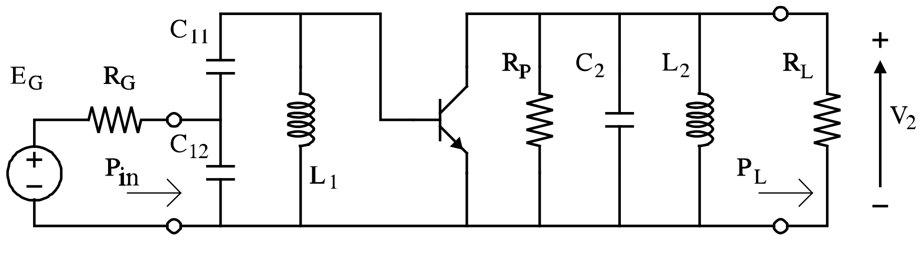


Fig.35

A transistor, which has the equivalent circuit in Fig.34 and components

$$r_\pi = 330\,[\Omega], \quad C_\pi = 15.\,[pF], \quad C_\mu = 0.5\,[pF], \quad g_m = 200.\,[mS],$$
$$C_o = 0.8\,[pF], \quad g_o = 0.3\,[mS]. \tag{136}$$

is used in a tuned amplifier, Fig.35. The amplifier must meet the specifications

- Center frequency, 100 MHz
- Synchronous tuning with totally 8 MHz 3dB bandwidth
- Simultaneous matching at input and output ports with R_G=50Ω, R_L to be found
- Gain as high as possible with K-factor = 5 in augmented estimations including R_G and R_L

Resistor R_p is included to meet the above requirements regarding K-factor and matching conditions.

- Find components C_{11}, C_{12}, L_1, R_p, C_2, L_2, and R_L
- Can amplifier stability be guaranteed if R_G, or R_L, or both are removed ?
- What is the center frequency gain P_L/P_{in} ?

P.III-3

Show that a strictly passive, reciprocal two-port - for instance one made of capacitors, inductors, and resistors - always is absolutely stable, and therefore always can be simultaneously matched at both ports.

P.III-4

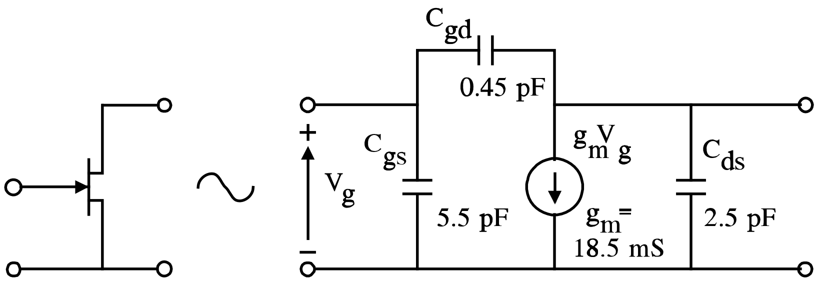


Fig.36

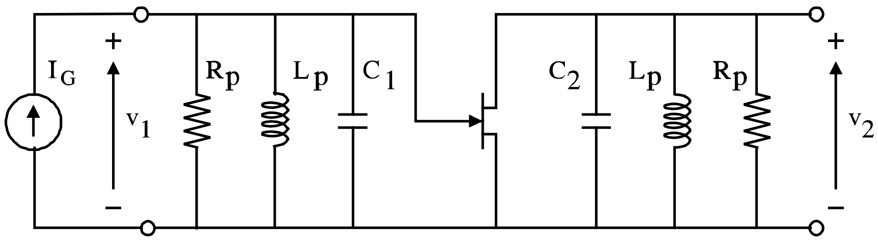


Fig.37

A FET that has the equivalent circuit in Fig.36 is used in the amplifier of Fig.37. Capacitors C_1 and C_2 are adjusted to make the input and output tuning circuits equal when the transistor capacitances are taken into account,

$$C_p \;=\; C_1 + C_{gs} + C_{gd} \;=\; C_2 + C_{ds} + C_{gd} \;=\; 20.\,[pF]\,.$$

Taken separately, for instance by short-circuiting the opposite port, the input and output resonance circuits are tuned to center frequency f_o=300 MHz.

- Find the greatest value R_p=R_{pmax} where the amplifiers stays stable and find the frequency of oscillation, if this values is slightly exceeded.
- Choose R_p to give a stability factor K = 5 for the complete setup. Calculate the center frequency magnitude and the 3 dB bandwidth of the corresponding trans-impedance, z=$|v_2/I_G|$.

P.III-5

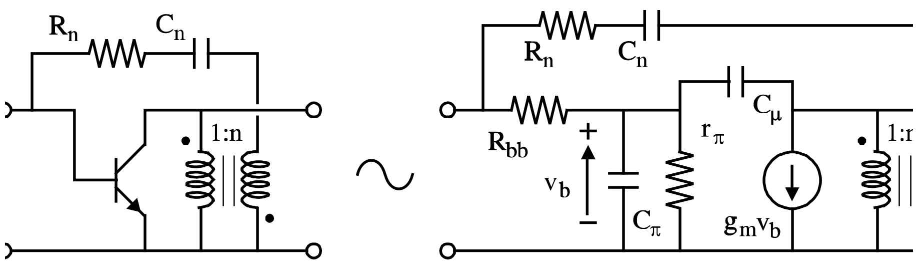


Fig.38 Neutralization of bipolar transistor with significant base series resistance.

Find expressions for the neutralizing components R_n and C_n in a bipolar transistor where the base series resistance is taken into account as shown in Fig.38.

Numerical example:

R_{bb}=50Ω, β=100, C_μ=2pF, f_T=1.5 GHz, n=0.5, DC current, I_C=10 mA.

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